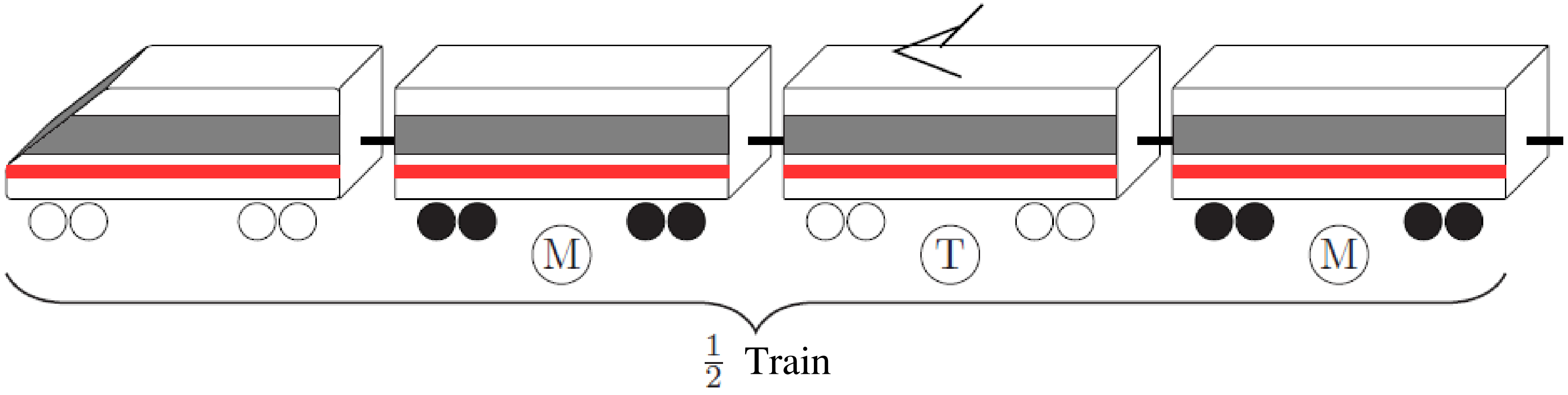
Superconducting Railway-Transformer

Design Example

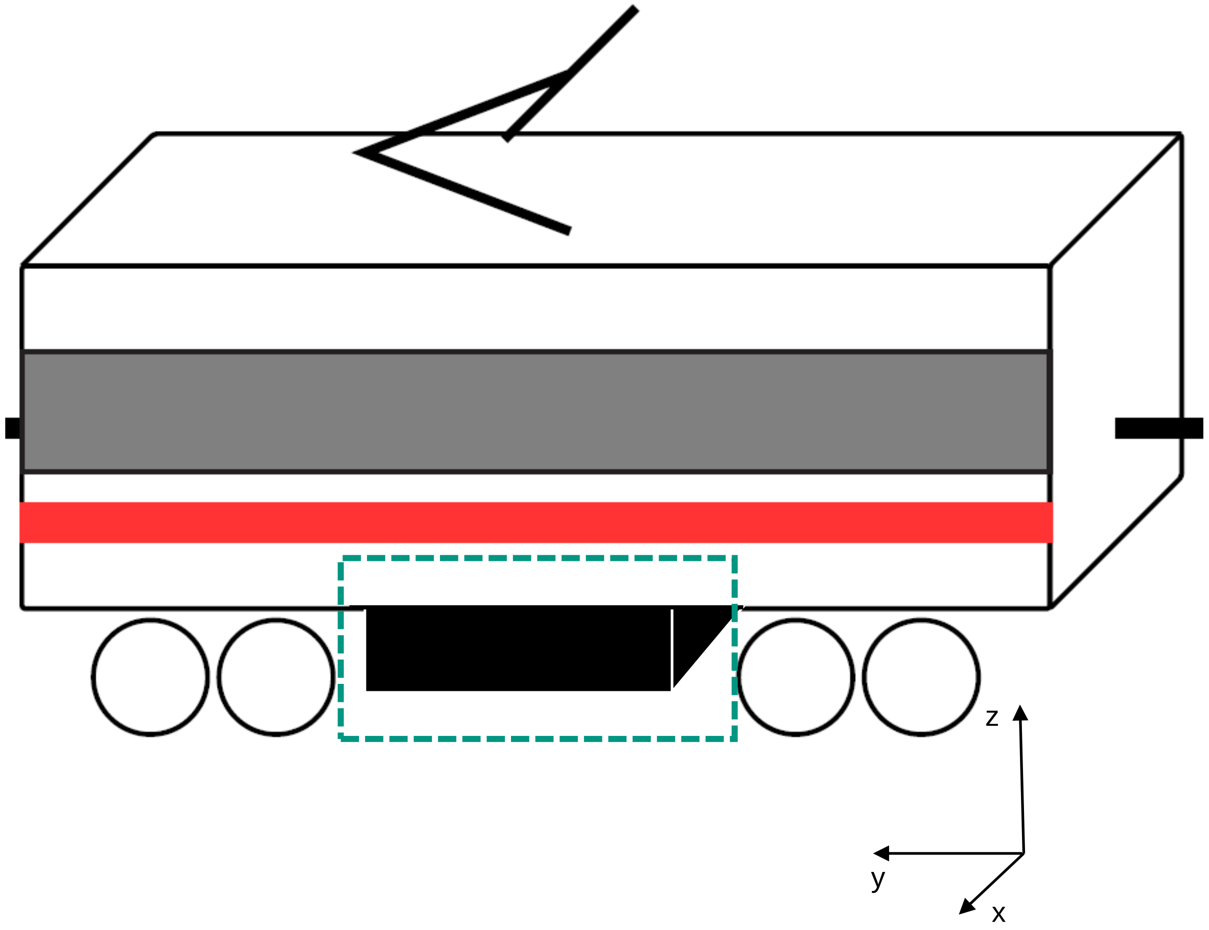
Example

- Rail route: Frankfurt (Main) – Cologne
  - Distance: 182 km
  - Top speed V_{max} = 300 km/h
  - Acceleration power P_{acc}= 8540 kW
- Train specs
  - 8 carriages per train, 4 powered, 4 passive
  - 2 transformers



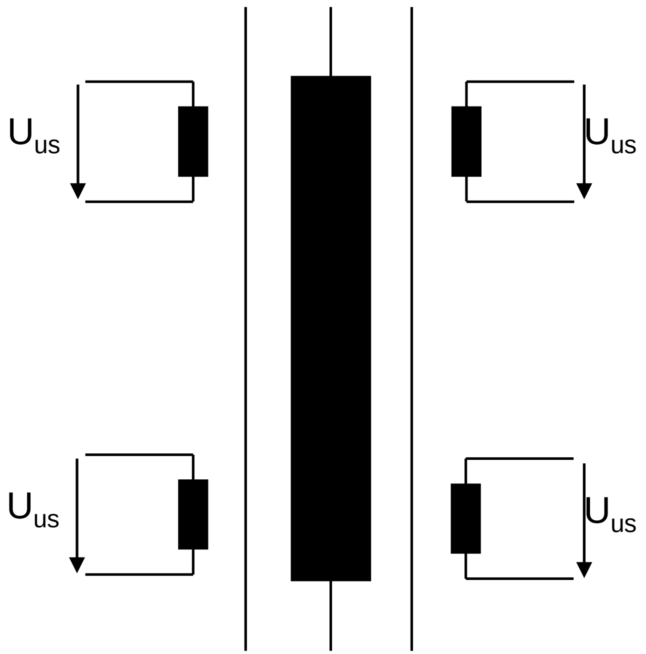
Transformer specs

- Maximum dimensions
  - Height
    - $h_{z,max} = 700mm$
  - Length
    - $h_{y,max} = 3400mm$
  - Width
    - $h_{x,max} = 2500mm$
  - Mass
    - $m_{max} = 6500kg$



Transformer specs

- Electrical data
  - Nominal power P_N
    - $P_N = 5000$ kW
  - Primary voltage U_{os}
    - $U_{os} = 25$kV ~ 50 Hz
  - Secondary voltage U_{us}
    - $U_{us} = 4 \times 1850$V (4 traction windings)
  - Relative short-circuit voltage u_k
    - $u_k \approx 40\%$



Driving Profile

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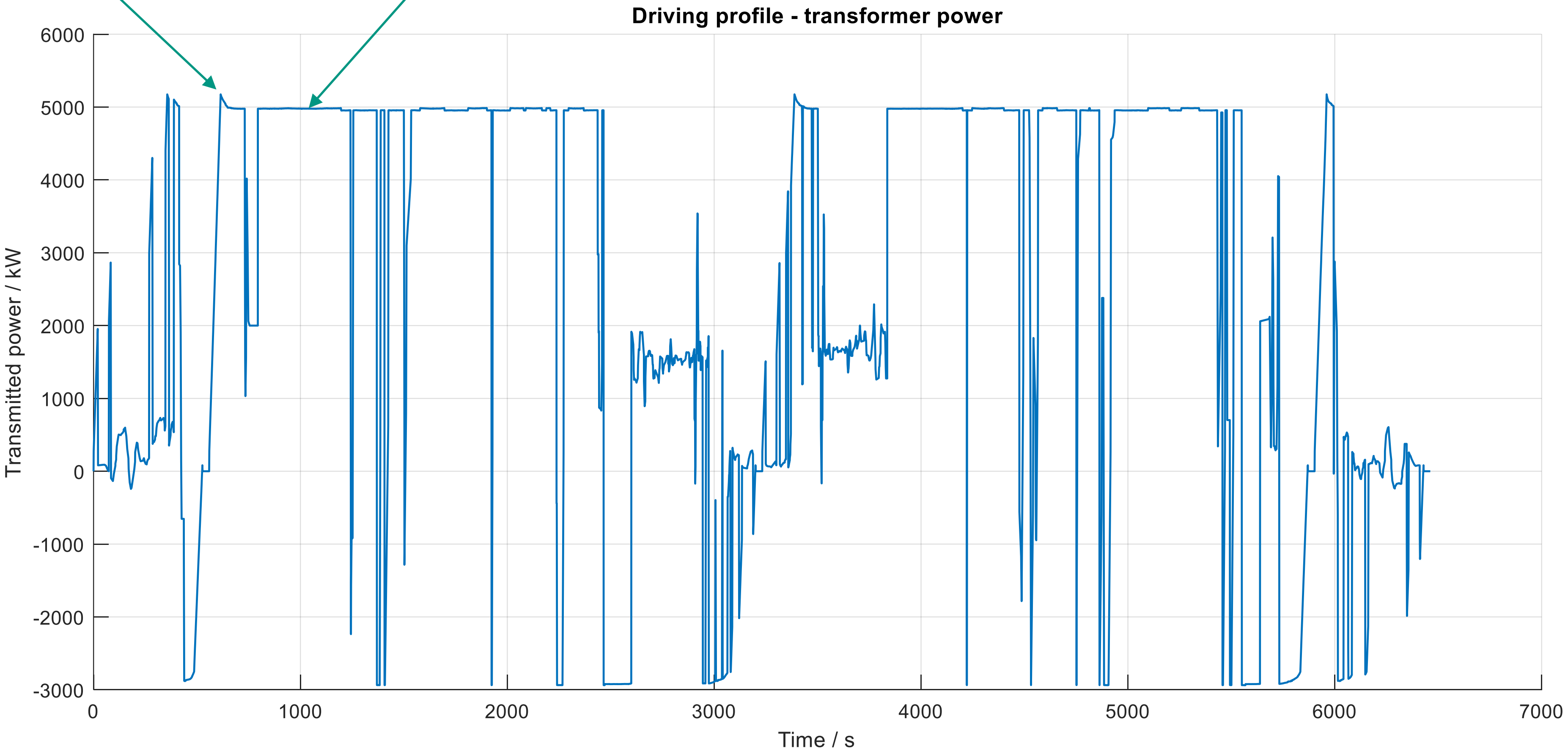
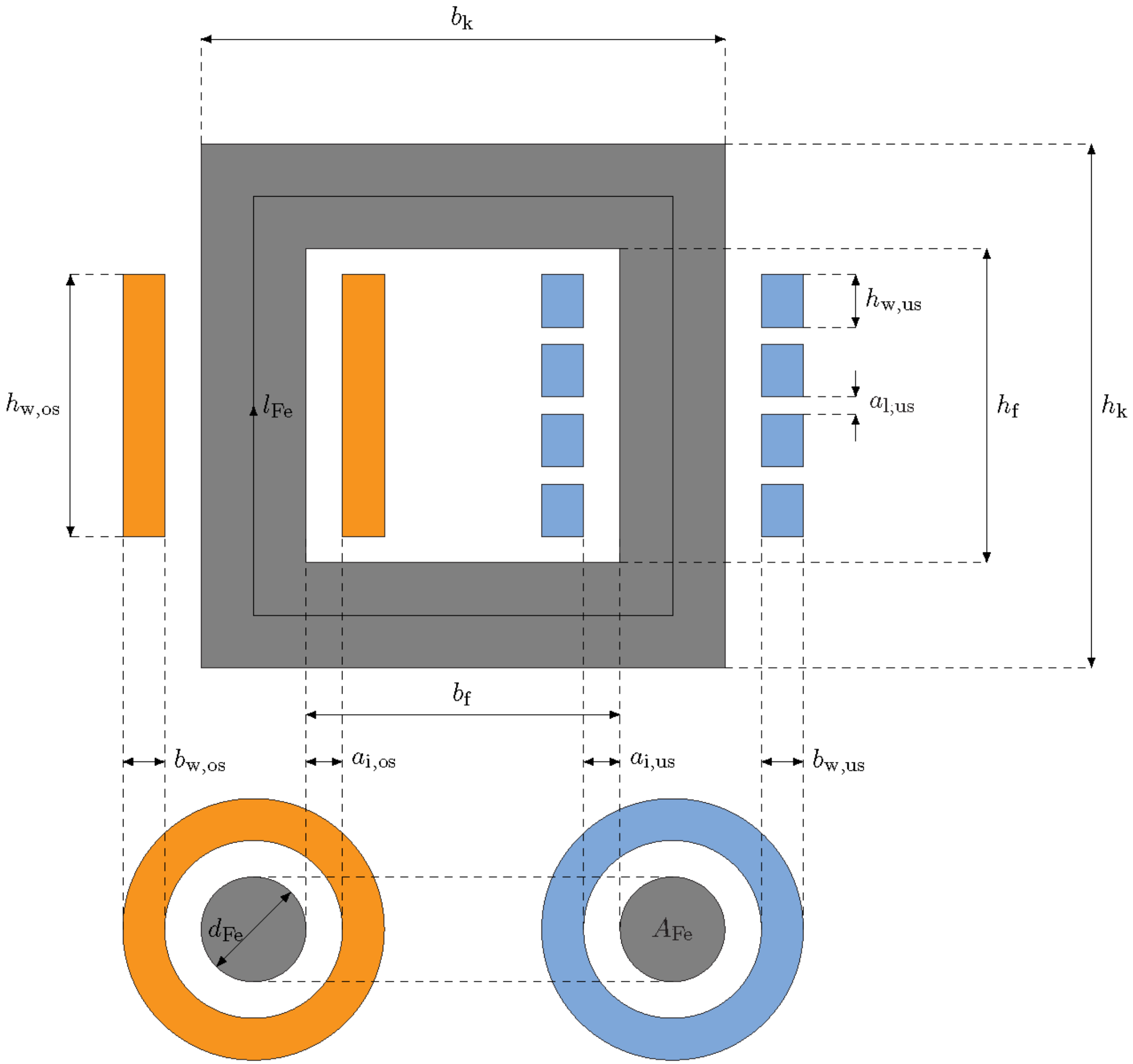
P_{max}=5175kW P_N=5000kW



Model geometry

Frequency	f	50 Hz
Nominal power (15 min. window of driving profile)	P_N	5000 kW
Maximum power	P_{max}	5175 kW
Rated power of superconducting windings (per winding)	$P_{r,sc} = \frac{1{,}05 * P_{max}}{4}$	1359 kW
Primary voltage	U_{os}	25 kV
Secondary voltage	U_{us}	4*1850 V
Rel. short-circuit voltage	u_k	40%
Amplitude of magnetic flux density	$\hat{B}_H$	1.5 T



Design process(1)

- Definition of optimization variables
  - Efficiency
  - Weight
  - Costs

When designing components for railway applications, it´s important to keep the costs as low as possible.
However, an increase in efficiency at the lowest possible cost is desirable.
→ **We choose the material costs as optimization variable**

Design process (2)

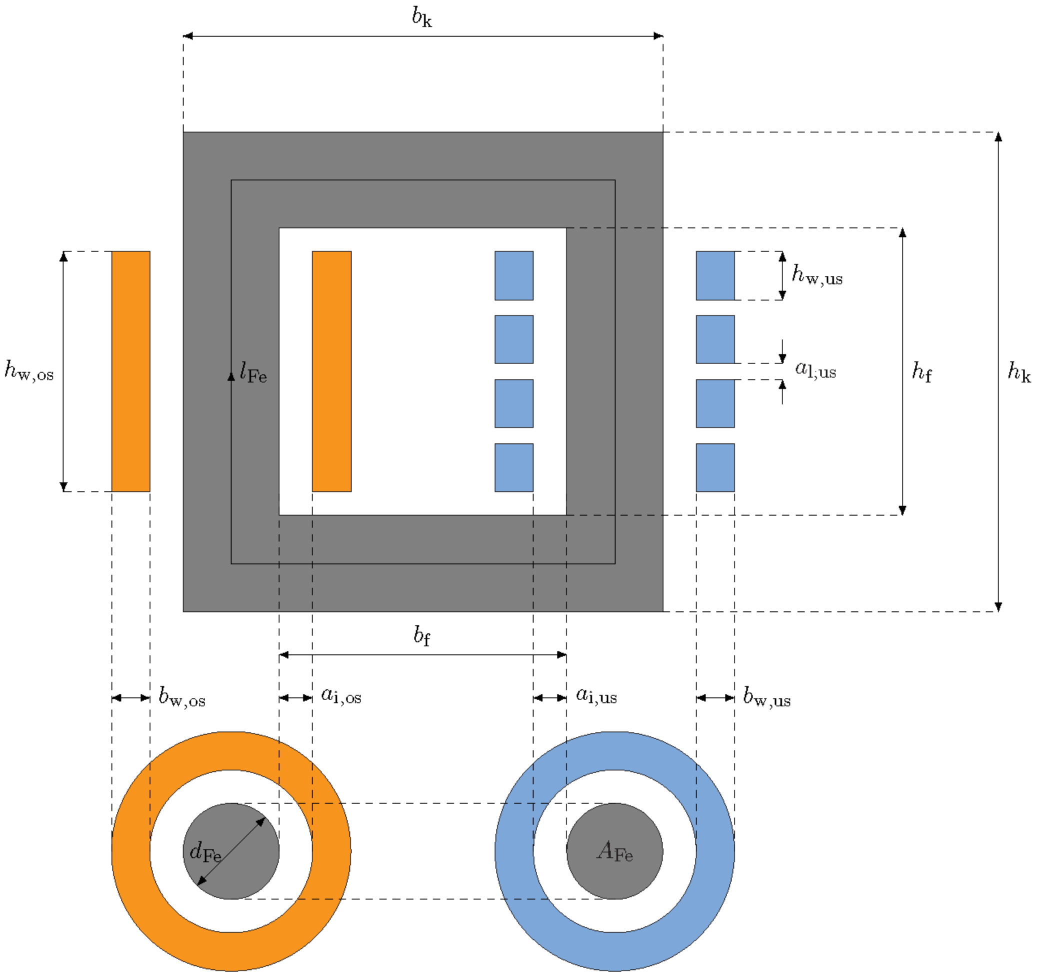
- Setting the range of the winding voltage 10V< u_w <50V (empirically)
  - Only winding voltages resulting in integer numbers of turns are considered.
  - Calculation of all other parameters in dependency of winding voltage u_w
  - Later: Adaptation of the initial range of winding voltages

- Calculating the number of turns w_{os} , w_{us}
  - $w_{os} = \frac{U_{os}}{u_w}$ $\quad Uos = 25\ kV$
  - $w_{us} = \frac{U_{us}}{u_w}$ $\quad U_{us} = 1850\ V$

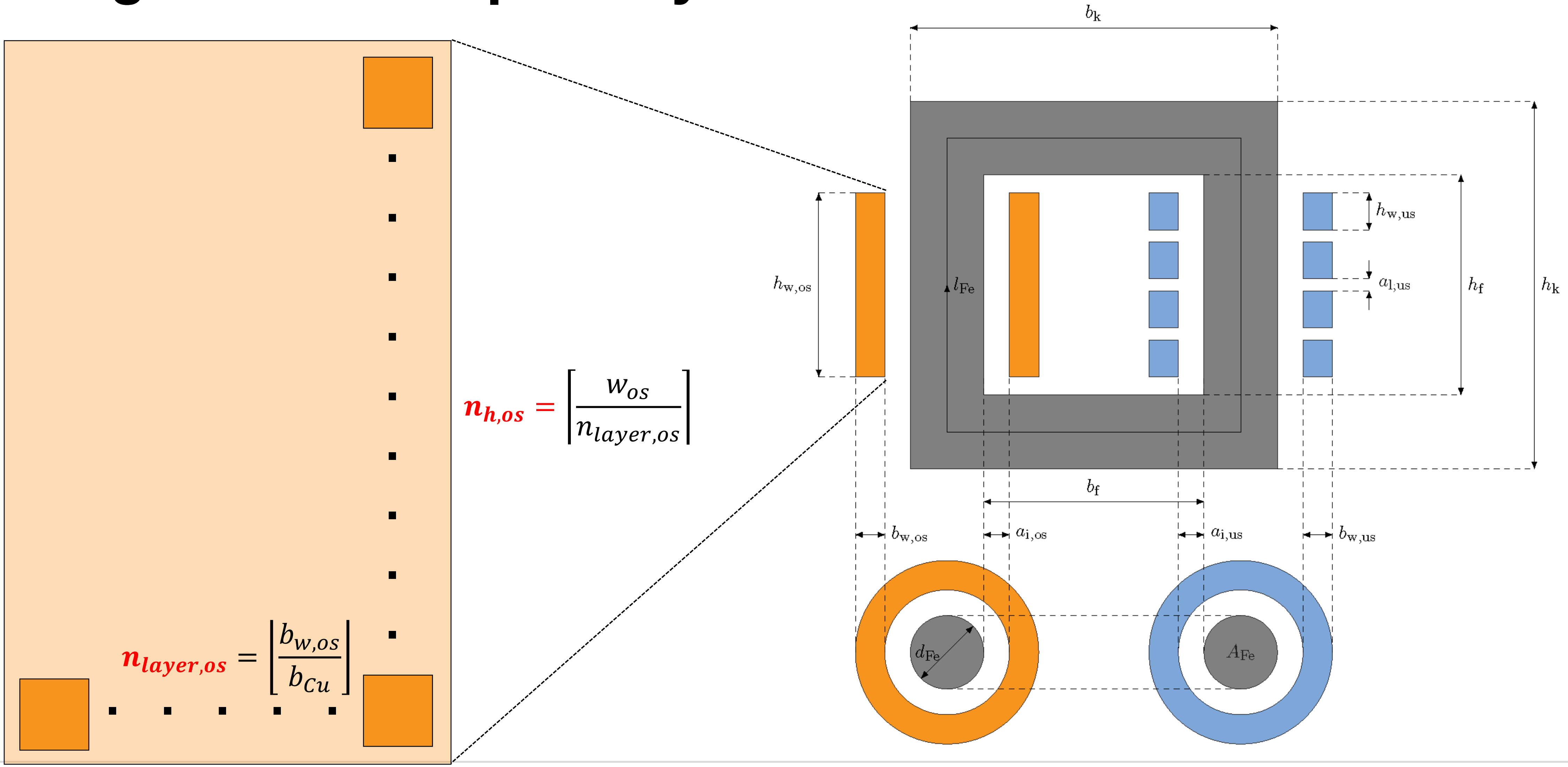
- Determination of the effective iron cross section
  - $A_{Fe,eff} = \frac{u_w}{\sqrt{2} * \pi * f * \widehat{B_H}}$

Design process (3)

- Determination of the winding structure – primary side
  - Calculating the conductor cross section $A_{Cu} = \frac{I_{os}}{j_{Cu}}$
    - Assumption: square shaped conductors
    - $h_{Cu} = b_{Cu} = \sqrt{A_{Cu}}$
  - **Maximizing the width of the primary windings to fit the maximum thickness of the transformer $h_{z,max}$=700mm**
    - $b_{w,os} = \frac{z_{max} - d_{Fe} - 2 * a_{i,os}}{2}$
  - Determination of layers
    - R-direction: $n_{layer,os} = \left\lceil \frac{b_{w,os}}{b_{Cu}} \right\rceil$
    - y-direction: $n_{h,os} = \left\lceil \frac{w_{os}}{n_{layer,os}} \right\rceil$



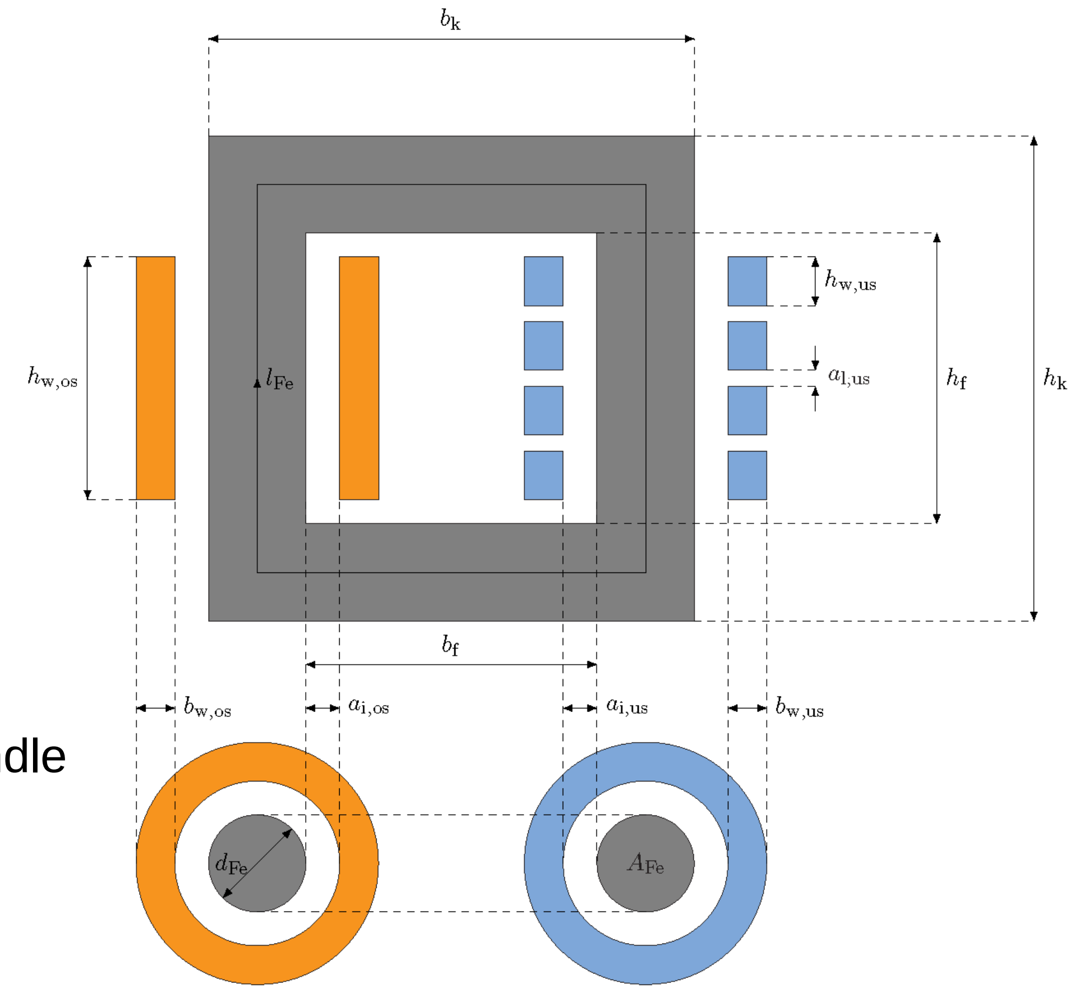
Winding structure – primary side



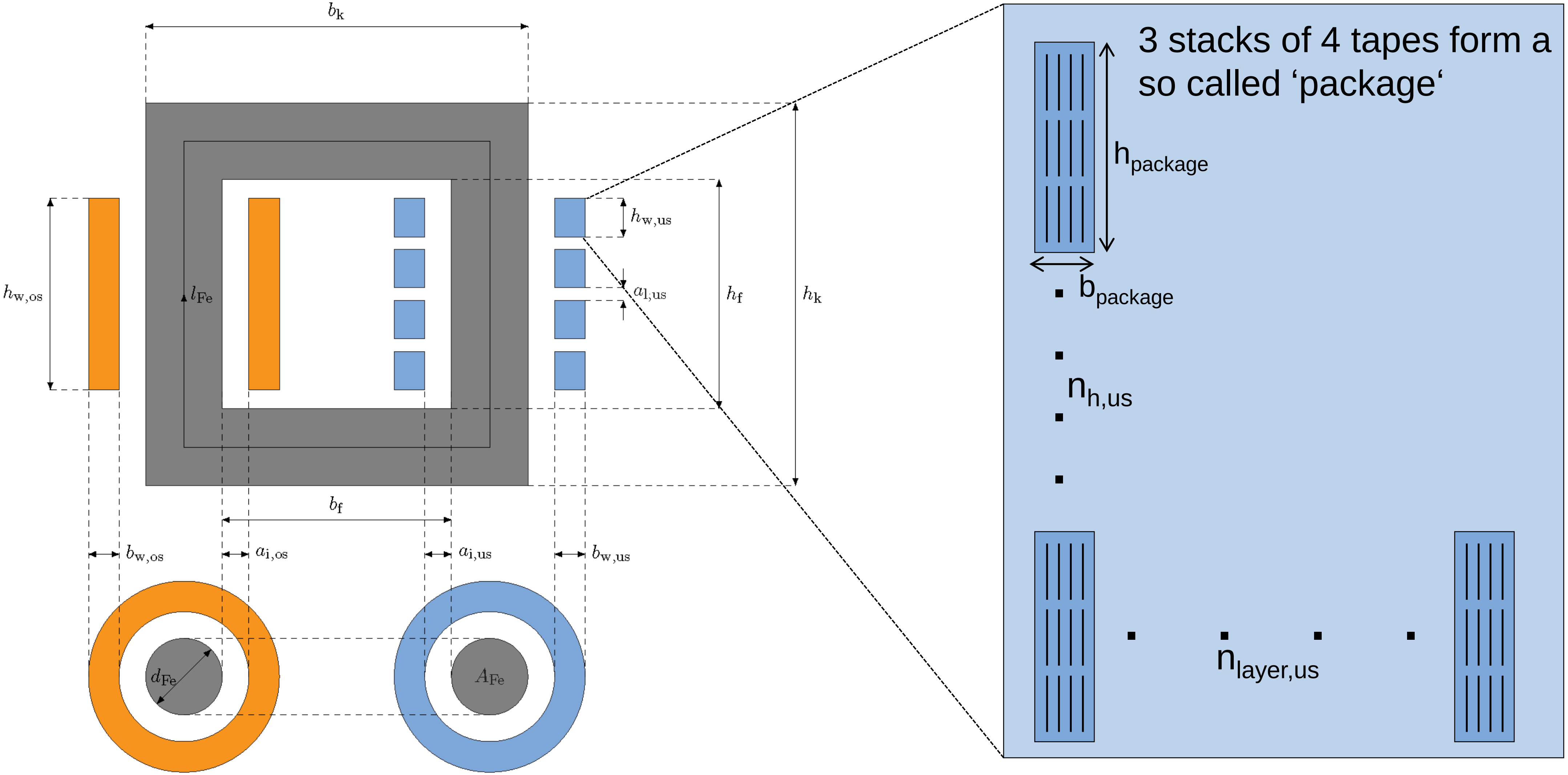
Design process (4)

Secondary side:

- Number of parallel tapes $\boldsymbol{n_{p,us}}$
  - According to the transformers stray-field, the critical current I_c is reduced up to 50%
    - I_c of modern tapes approx. 200A (width = 4mm)
    - $I_{c,red} \approx$ 100A
  - $n_{p,us} = \frac{\sqrt{2} * P_{b,sc}}{U_{us} * I_{c,red}} = \frac{\sqrt{2} * 1359kW}{1850V * 100A} = 10{,}39$
    - A minimum of 11 parallel tapes is needed
      - Stacks of 11 tapes are not easy to fabricate/handle
      - We choose $\boldsymbol{n_{p,us} = 12}$
      - 3 stacks of 4 tapes each

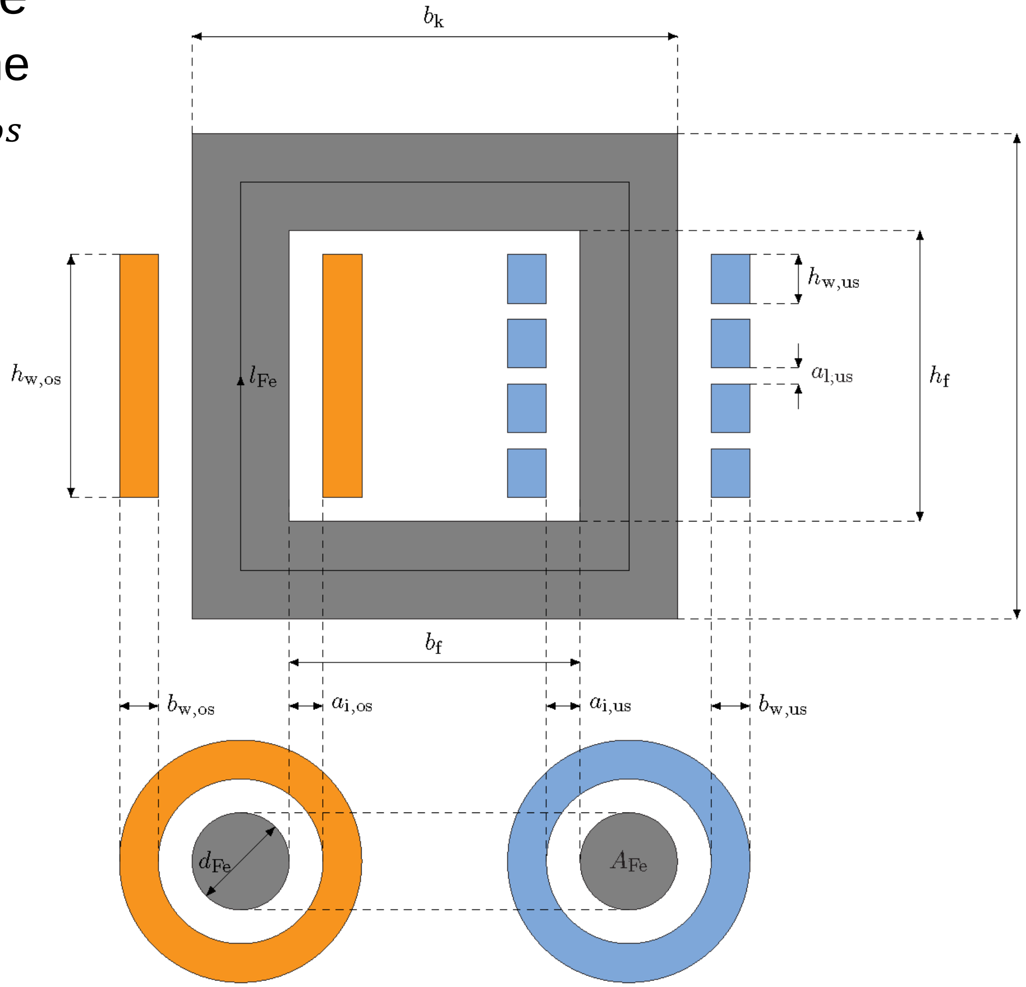


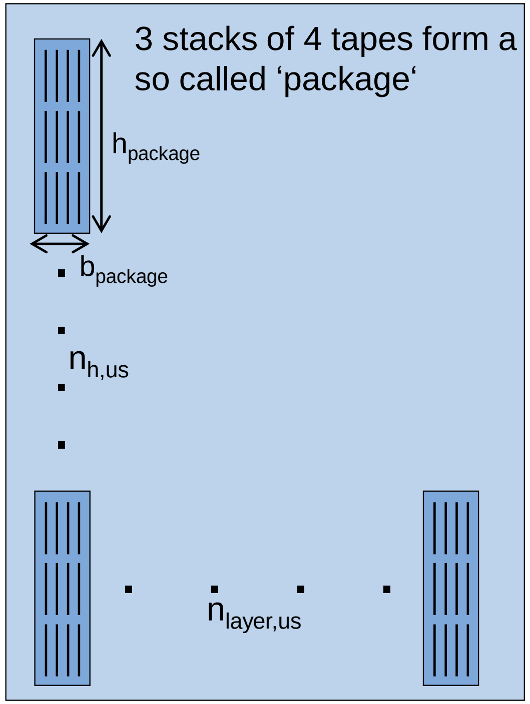
Winding structure – secondary side



Design process (5)

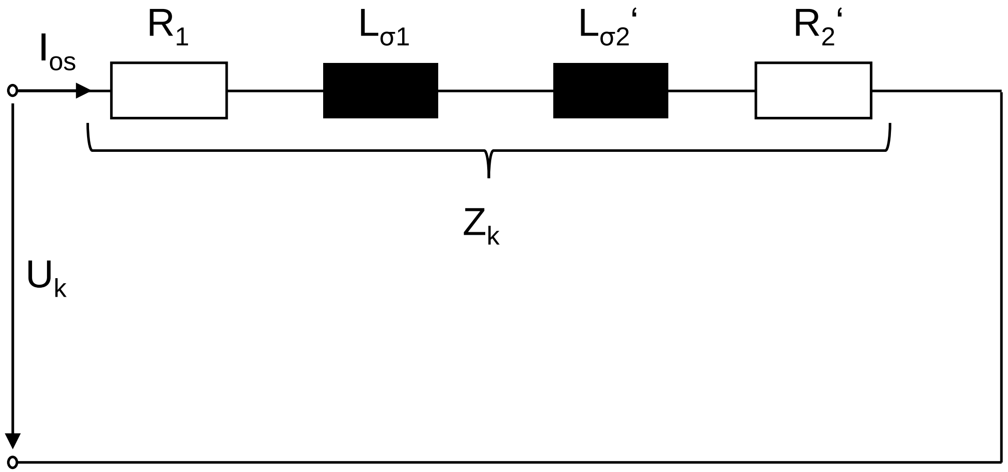
- Winding structure – secondary side
  - Height of windings has to match the height of the primary windings $h_{w,os}$
    - $h_{w,us} = \frac{h_{w,os} - 3 * a_{l,us}}{4}$
  - Stacking the winding packages
    - y-direction: $\boldsymbol{n_{h,us}} = \left\lfloor \frac{h_{w,us}}{w_{us}} \right\rfloor$
    - R-direction: $\boldsymbol{n_{layer,us}} = \left\lfloor \frac{w_{us}}{n_{h,us}} \right\rfloor$





Design process (6)

- Calculating the stray inductance L_σ from the relative short-circuit voltage u_k
  - $\mathbf{L_\sigma = L_{\sigma 1} + L'_{\sigma 2}}$
  - $Z_k = \frac{U_k}{I_{os}} = j\omega L_\sigma + R_1 + R_2'$
  - $u_k = \frac{U_k}{U_{os}} = (j\omega L_\sigma + R_1 + R_2') * \frac{I_{os}}{U_{os}} \overset{!}{=} 0.4$
  - $L_\sigma = \frac{u_{k*} U_{os}}{j\omega * I_{os}} - \underbrace{\left(\frac{(R_1 + R_2')}{j\omega}\right)}_{ignored}$



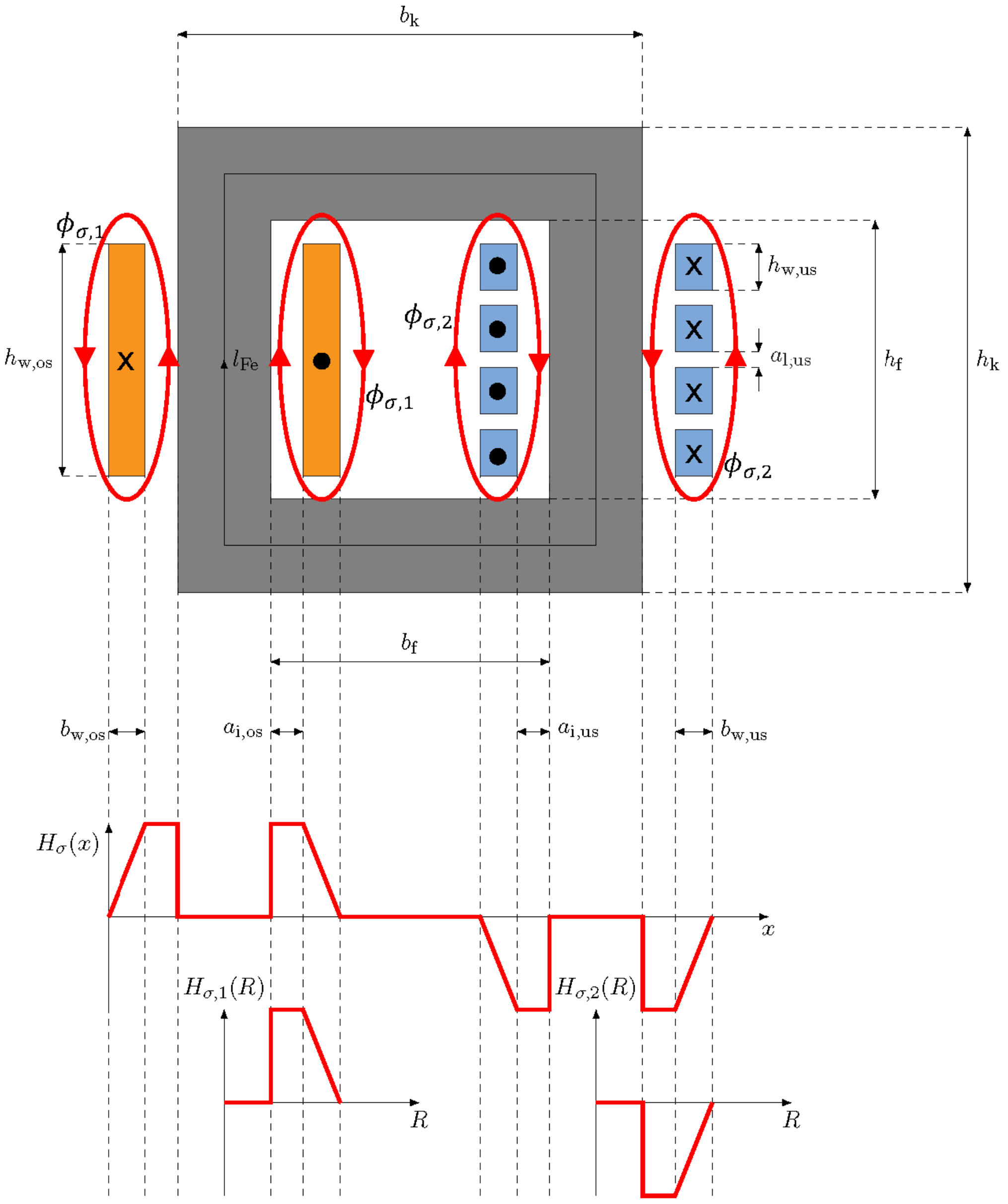
Short-circuit equivalent circuit diagram

Design process (7)

- Calculating the Energy of the magnetic stray field
  - $\boldsymbol{W_{mag,\sigma} = \frac{1}{2} L_\sigma I^2 = \frac{1}{2} \iiint_V B_\sigma H_\sigma \, dV = \frac{1}{2} \mu_0 \iiint_V {H_\sigma}^2 \, dV}$
- Making use of cylindrical symmetries

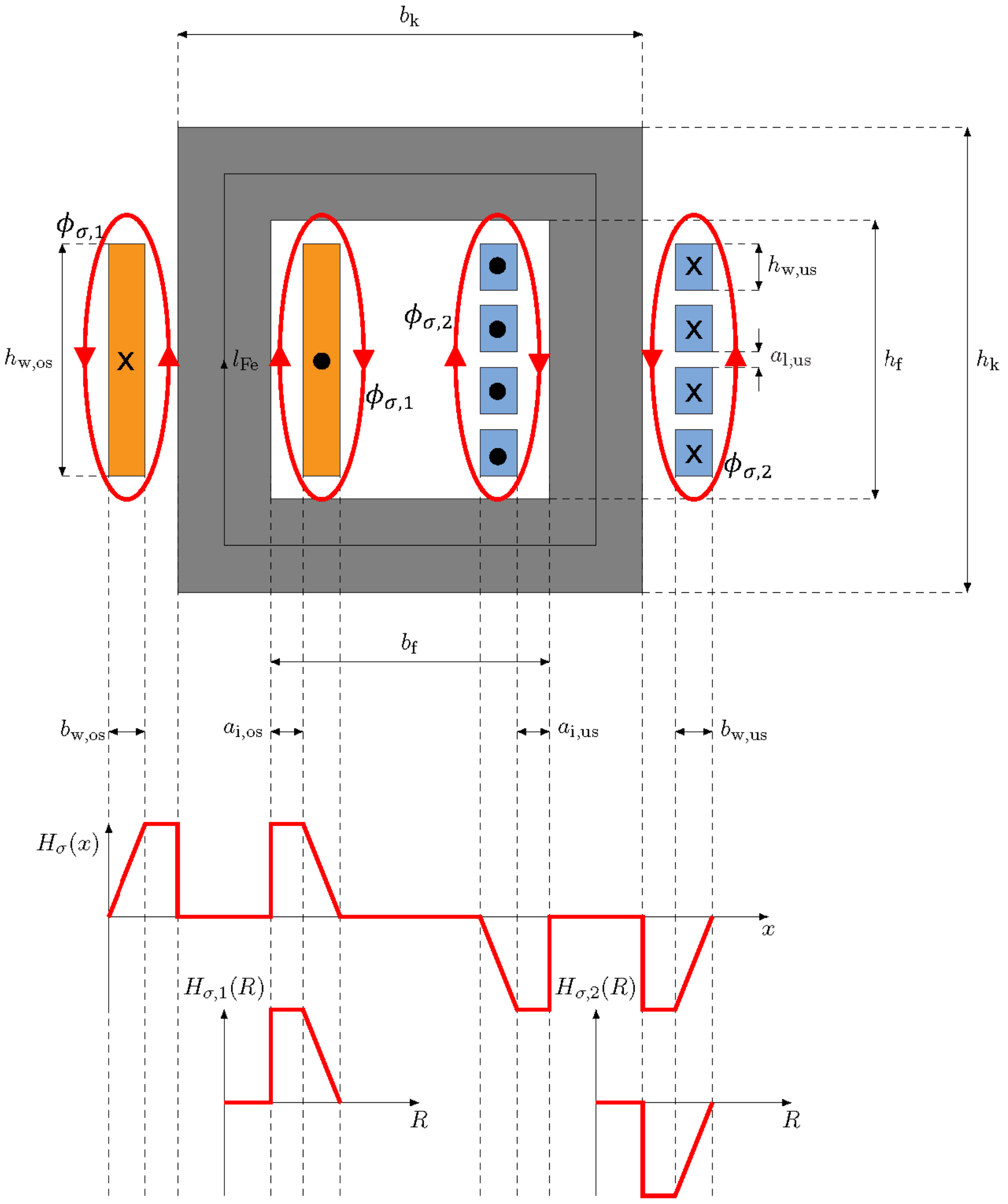
$$H_{\sigma,1}(R) = \begin{cases} \frac{w_{os} * I_{os}}{h_{w,os}} & , \frac{d_{Fe}}{2} \leq R \leq \frac{d_{Fe}}{2} + a_{i,os} \\ \frac{w_{os} * I_{os}}{h_{w,os}} * \frac{\frac{d_{Fe}}{2} + a_{i,os} + b_{os} - R}{b_{os}} & , \frac{d_{Fe}}{2} + a_{i,os} \leq R \leq \frac{d_{Fe}}{2} + a_{i,os} + b_{os} \end{cases}$$

$$H_{\sigma,2}(R) = \begin{cases} -\frac{w_{us} * I_{us}}{4 * h_{w,us}} & , \frac{d_{Fe}}{2} \leq R \leq \frac{d_{Fe}}{2} + a_{i,us} \\ -\frac{w_{us} * I_{us}}{4 * h_{w,os}} * \frac{\frac{d_{Fe}}{2} + a_{i,us} + b_{us} - R}{b_{us}} & , \frac{d_{Fe}}{2} + a_{i,us} \leq R \leq \frac{d_{Fe}}{2} + a_{i,us} + b_{us} \end{cases}$$



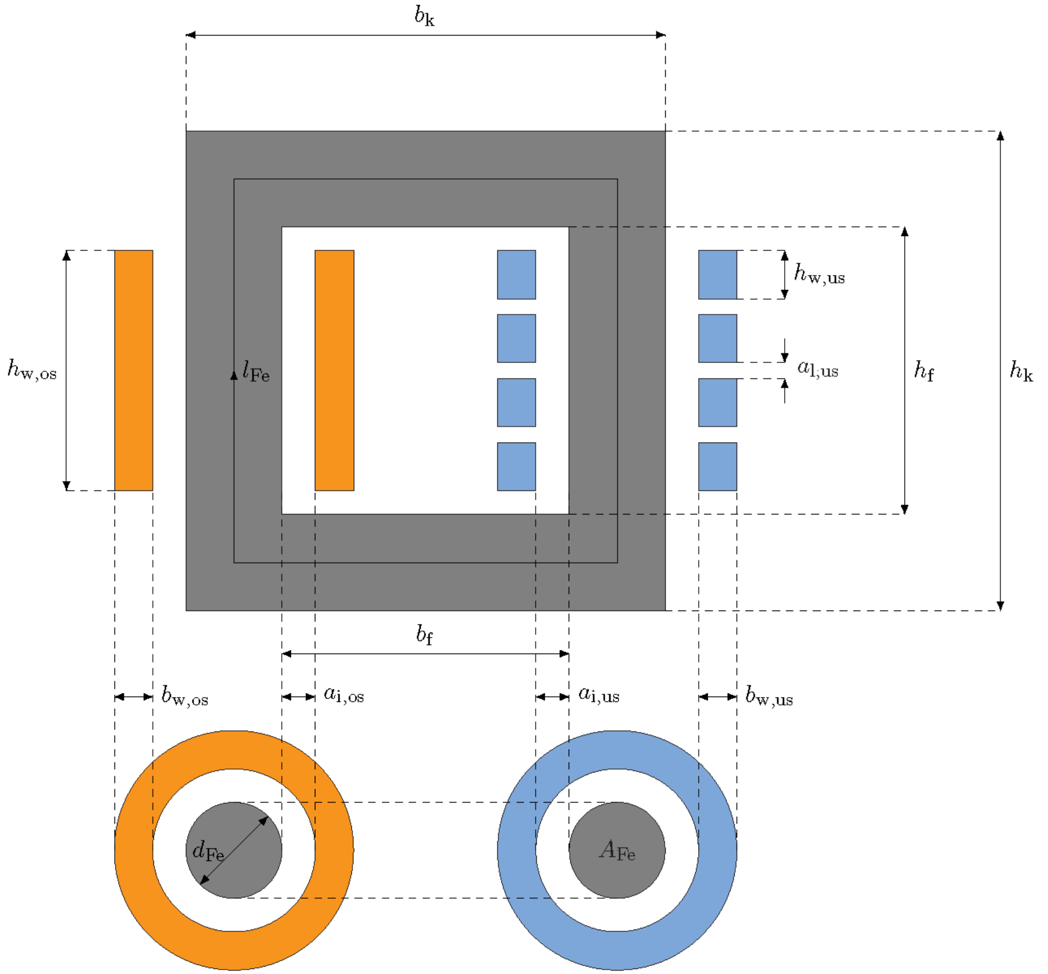
Design process (8)

- Calculating the Energy of the magnetic stray field
  - $W_{mag,\sigma} = \frac{1}{2} L_\sigma I^2 = \frac{1}{2} \iiint_V B_\sigma H_\sigma \, dV = \frac{1}{2} \mu_0 \iiint_V {H_\sigma}^2 \, dV$
- Integration:
  - $L_{\sigma,1} = \frac{\mu_0 * {w_{os}}^2}{h_{w,os}} * 2\pi * \left[\frac{{a_{i,os}}^2}{2} + a_{i,os} * \left(\frac{d_{Fe}}{2} + \frac{b_{w,os}}{3} \right) + \frac{{b_{w,os}}^2}{12} + b_{os} * \frac{d_{Fe}}{6} \right]$
  - $L_{\sigma,2} = \frac{\mu_0 * {w_{us}}^2}{4 * h_{w,us}} * 2\pi * \left[\frac{{a_{i,us}}^2}{2} + a_{i,us} * \left(\frac{d_{Fe}}{2} + \frac{b_{w,us}}{3} \right) + \frac{{b_{w,us}}^2}{12} + b_{us} * \frac{d_{Fe}}{6} \right]$
  - $L_{\sigma,1}$ und $L_{\sigma,2}$ can be calculated from u_k
- Calculating the needed air gaps $a_{i,os}$ und $a_{i,us}$
  - Solving quadratic equations
  - $a_{i,os} = -\left(\frac{d_{Fe}}{2} + \frac{b_{w,os}}{3} \right) \pm \sqrt{\left(\frac{d_{Fe}}{2} + \frac{b_{w,os}}{3} \right)^2 - \frac{{b_{w,os}}^2}{6} - \frac{b_{w,os} * d_{Fe}}{3} + \frac{L_{\sigma,1} * h_{w,os}}{\mu_0 * {w_{os}}^2 * \pi}}$
  - $a_{i,us} = -\left(\frac{d_{Fe}}{2} + \frac{b_{w,us}}{3} \right) \pm \sqrt{\left(\frac{d_{Fe}}{2} + \frac{b_{w,us}}{3} \right)^2 - \frac{{b_{w,us}}^2}{6} - \frac{b_{w,us} * d_{Fe}}{3} + \frac{L_{\sigma,2} * 4 * h_{w,us}}{\mu_0 * {w_{us}}^2 * \pi}}$



Design process (9)

- Calculation of dimensions and mass of the iron core
  - Window height h_f and -width b_f
    - $h_f \approx h_{w,os}$
    - $b_f \approx b_{w,os} + b_{w,us}$
  - Core height h_k and -width b_k
    - $h_k \approx h_f + 2 * d_{Fe}$
    - $b_k \approx b_f + 2 * d_{Fe}$
  - Volume of the iron core V_{Fe}
    - $V_{Fe} = A_{Fe,eff} * 2 * \left(h_f + b_k\right)$
  - Mass of the iron core m_{Fe}
    - $m_{Fe} = V_{Fe} * \rho_{Fe}$



Design process (10)

- Calculation of <u>(copper) conductor length , -volume and -mass (primary side)</u>
  - Making use of mean winding diameter and the number of primary turns w_{os}
    - $l_{Cu} = w_{os} * \pi * \left[d_{Fe} + \left(n_{layer,os} * b_{Cu}\right) + 2 * a_{i,os}\right]$
    - $V_{Cu} = l_{Cu} * A_{Cu}$
    - $m_{Cu} = V_{Cu} * \rho_{Cu}$

- Calculation of <u>total length of superconductor</u>
  - Making use of mean winding diameter and the number of secondary turns w_{us}
    - $l_{sc} = 4 * n_{p,us} * w_{os} * \pi * \left[d_{Fe} + \left(n_{layer,us} * b_{Cu}\right) + 2 * a_{i,us}\right]$

Design process (11)

- Calculating copper resistance
  - $R_{Cu} = \rho_{Cu} * \frac{l_{Cu}}{A_{Cu}}$
- Calculating copper and iron losses
  - $P_{v,Cu} = R_{Cu} * {I_{os}}^2$
  - $P_{v,Fe} = v_{Fe} * m_{Fe}$
- Calculating the efficiency (without ac-losses)
  - $\eta = 1 - \frac{P_{v,Cu}+P_{v,Fe}}{P_N}$
- Calculating the costs
  - $K_{Fe} = k_{Fe} * m_{Fe}$
  - $K_{Cu} = k_{Cu} * m_{Cu}$
  - $K_{sc} = k_{sc} * l_{sc}$
  - $K_{tot} = K_{Fe} + K_{Cu} + K_{sc}$

$k_{Fe} = 9 \frac{€}{kg}$

$k_{Cu} = 15 \frac{€}{kg}$

$k_{sc} = 20 \frac{€}{m}$

Specific material costs

Design process (12)

- Finding the optimal design
  - Target: Minimal material costs
    - $K_{Fe} = k_{Fe} * m_{Fe}$
    - $K_{Cu} = k_{Cu} * m_{Cu}$
    - $K_{sc} = k_{sc} * l_{sc}$
    - $K_{tot} = K_{Fe} + K_{Cu} + K_{sc}$

$$k_{Fe} = 9\frac{€}{kg}$$
$$k_{Cu} = 15\frac{€}{kg}$$
$$k_{sc} = 20\frac{€}{m}$$

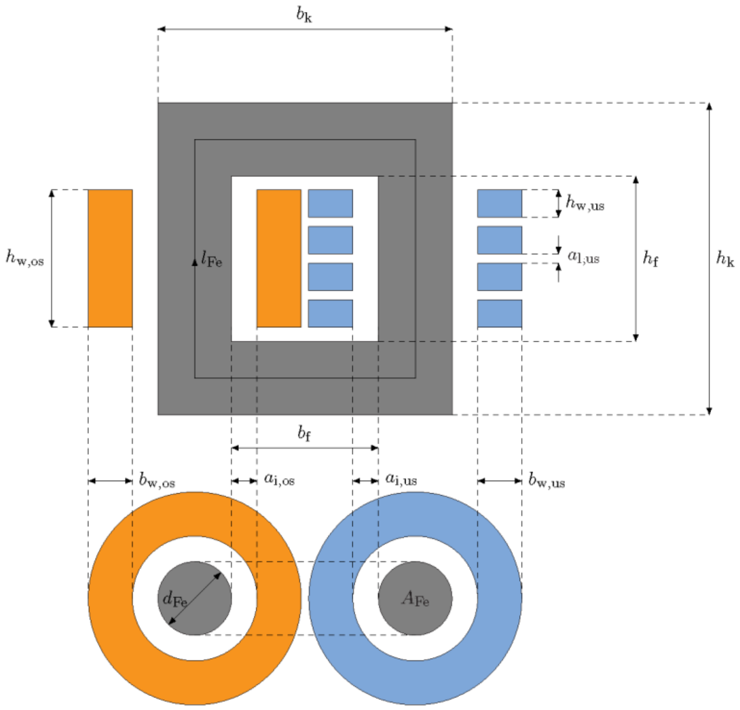
Specific material costs

→K_{tot} has to be minimized

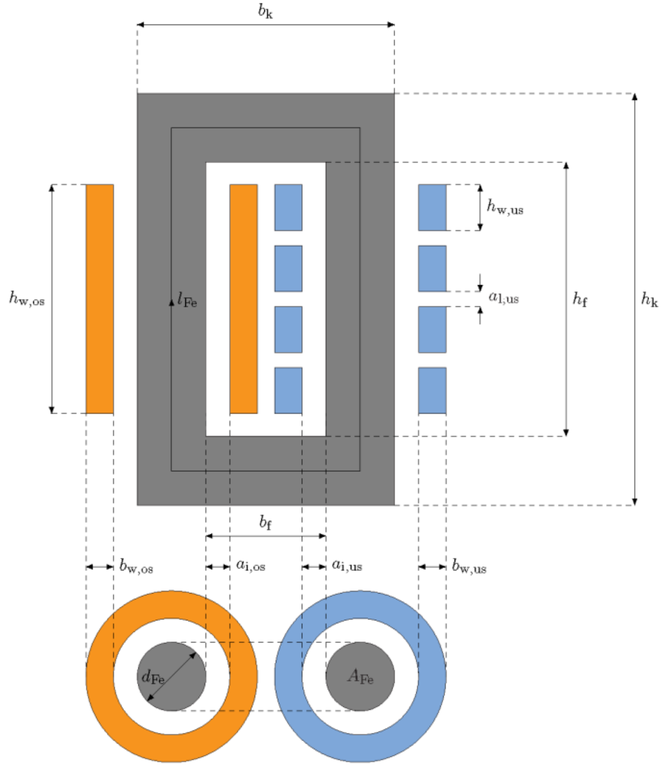
Short insertion – winding ratio a_v

- Winding ratio $a_v = \frac{h_{w,os}}{b_{w,os}}$ has a big influence on the transformers shape

- Small values of a_v
  - Transformer appears rather dense and short



- Big values of a_v
  - Transformer appears rather slim and long

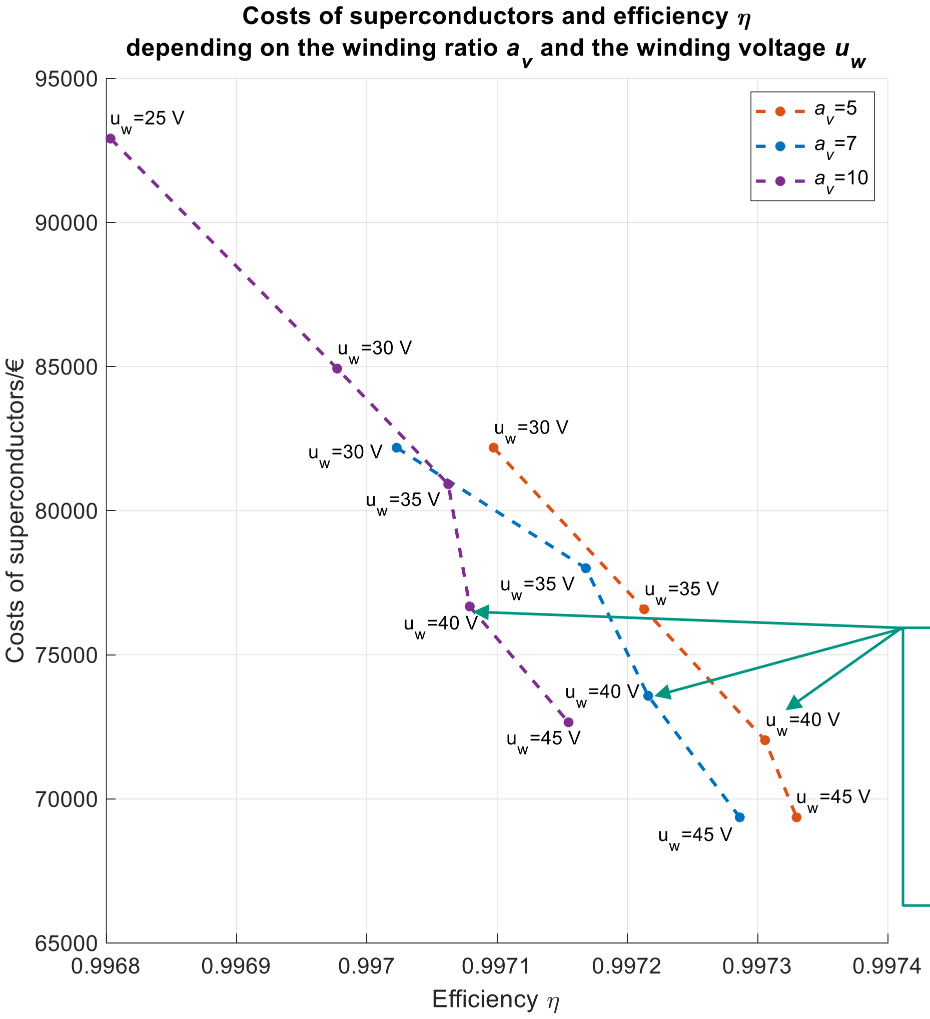


Short insertion – winding ratio a_v

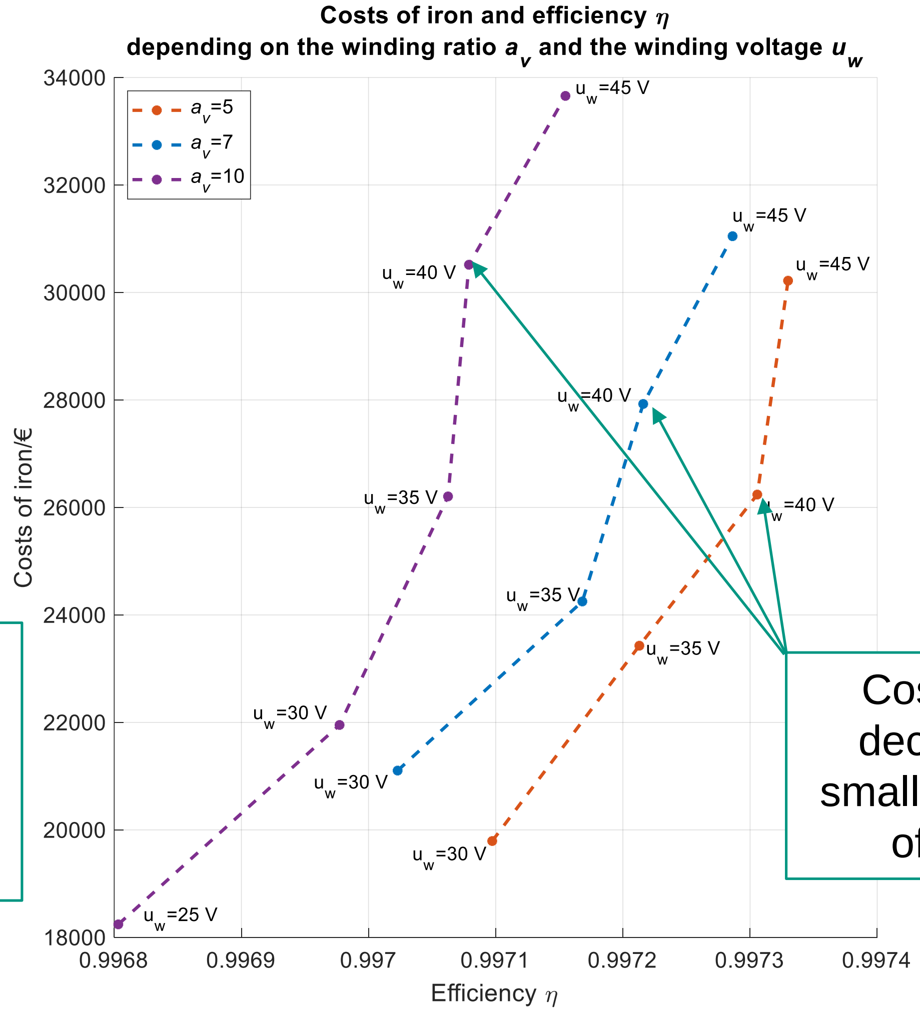
- How does the winding ratio a_v influence the optimization variables?
  - Influence on costs of superconductors?
  - Influence on costs of iron?
  - Influence on total costs?

- Additionally:
  - Influence on efficiency η?

Impact of a_v on costs and efficiency

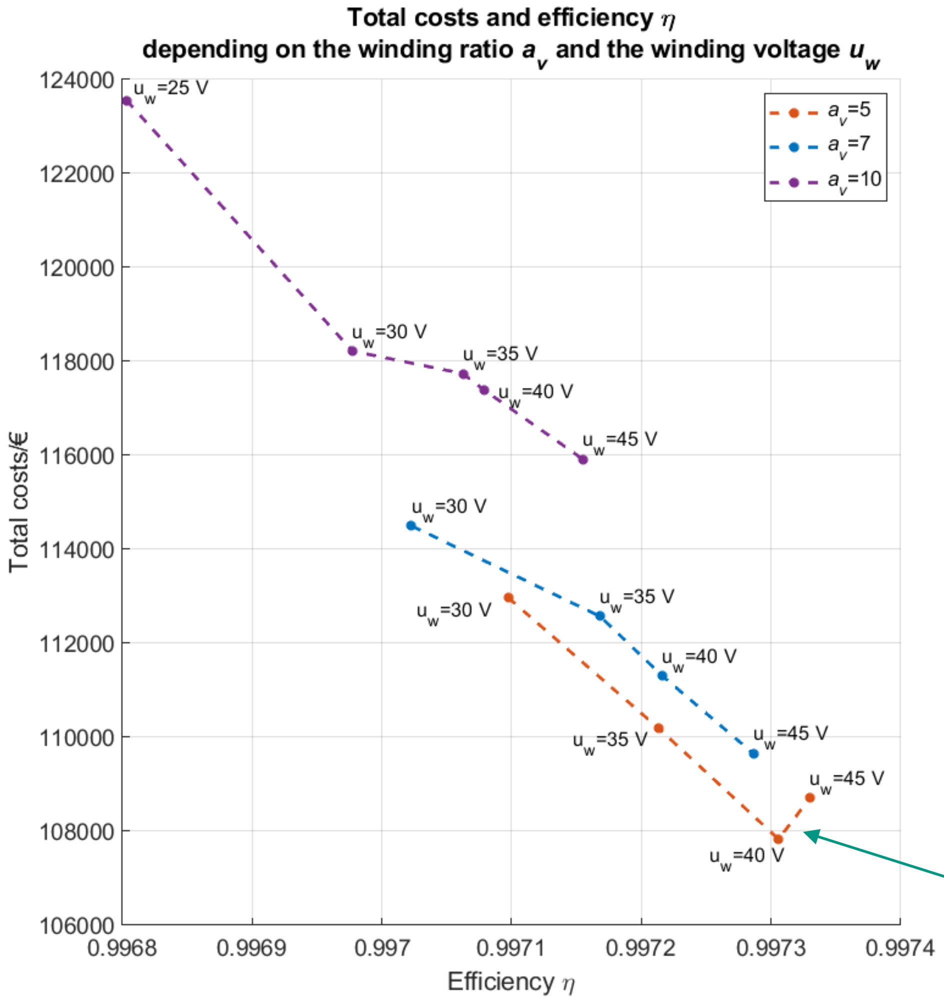


Costs of superconductors decrease the smaller the value of a_v gets



Costs of iron decrease the smaller the value of a_v gets

Impact of a_v on costs and efficiency

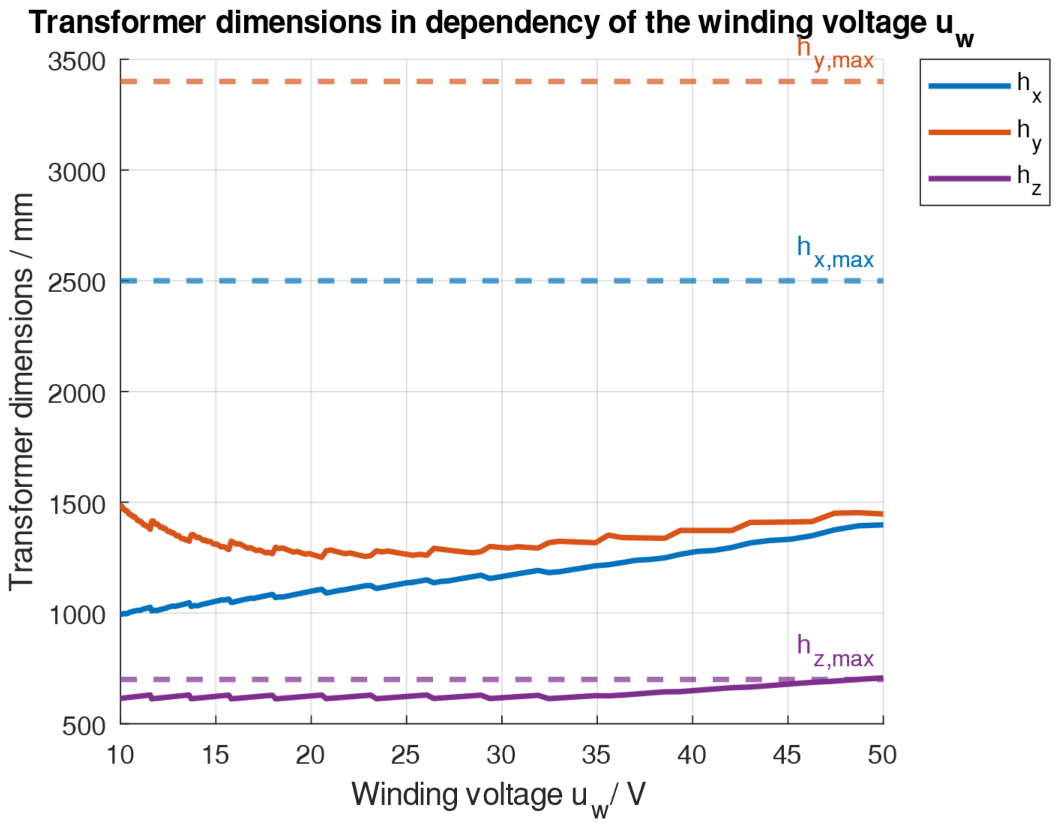


- Conclusion:
  - The smaller the value a_v the cheaper and more efficient the transformer will be.
  - High winding voltages u_w result in low superconductor quantity (and copper quantity) but high iron cross section A_{Fe}
    - Cost development of superconductor and iron is opposite in terms of winding voltage
    - Total costs are the decisive factor
    - Tendency to "stubby" design

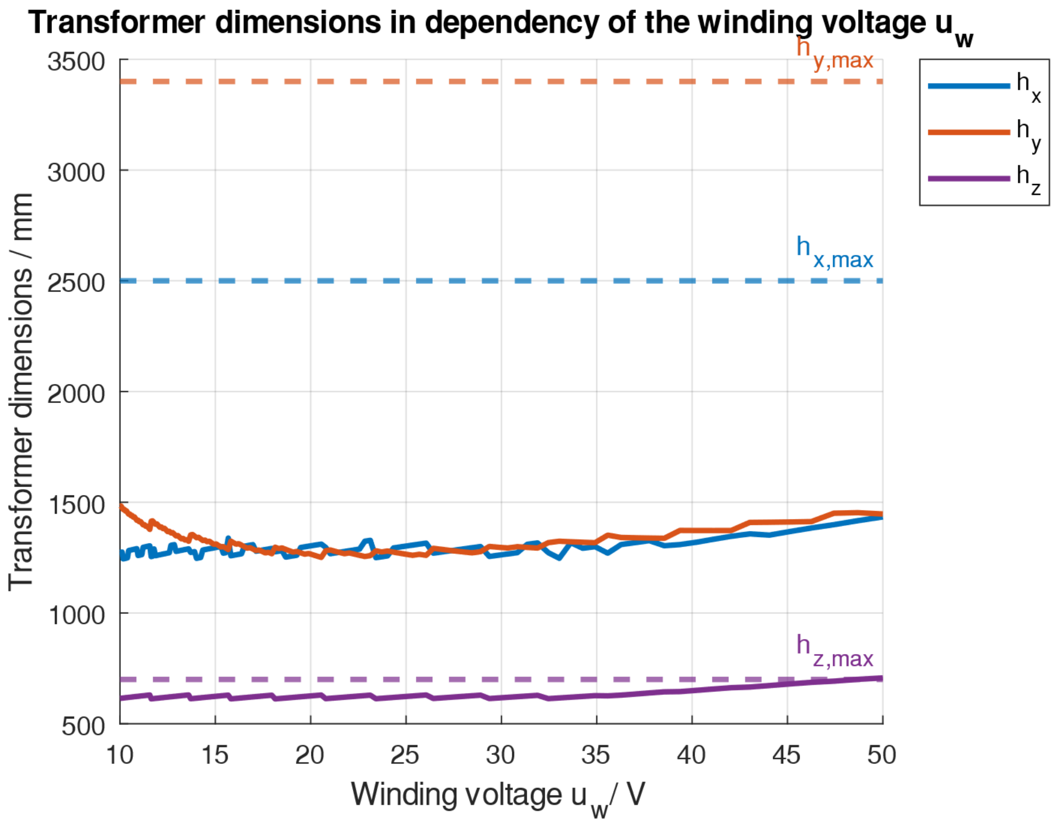
Lowest total costs & relatively high efficiency

Results of the design process (1)

superconducting

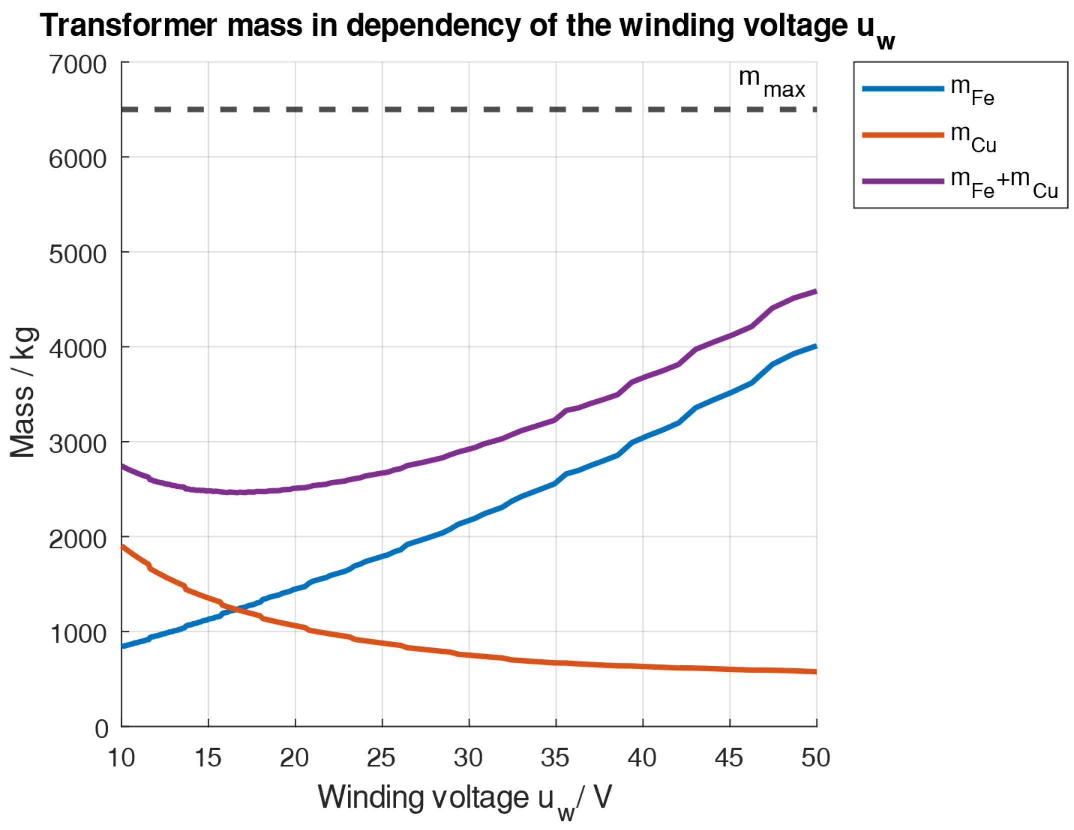


normal conducting

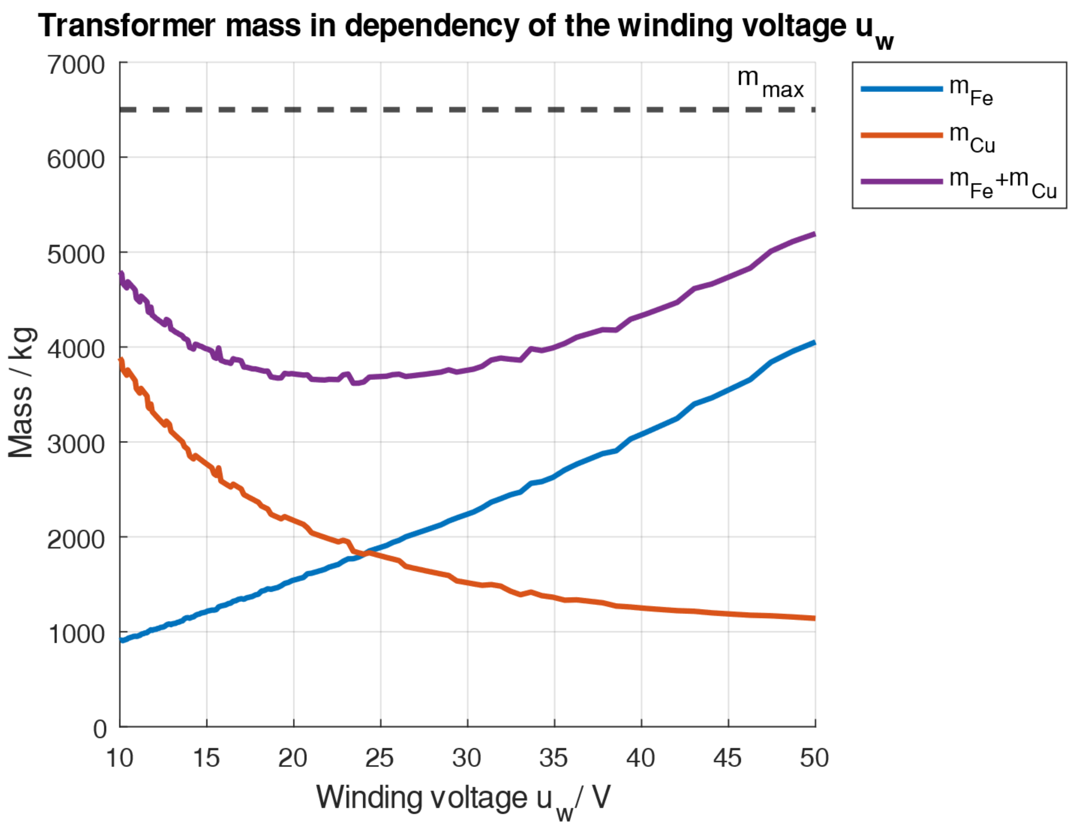


Results of the design process (2)

superconducting



normal conducting



Results of the design process (3)

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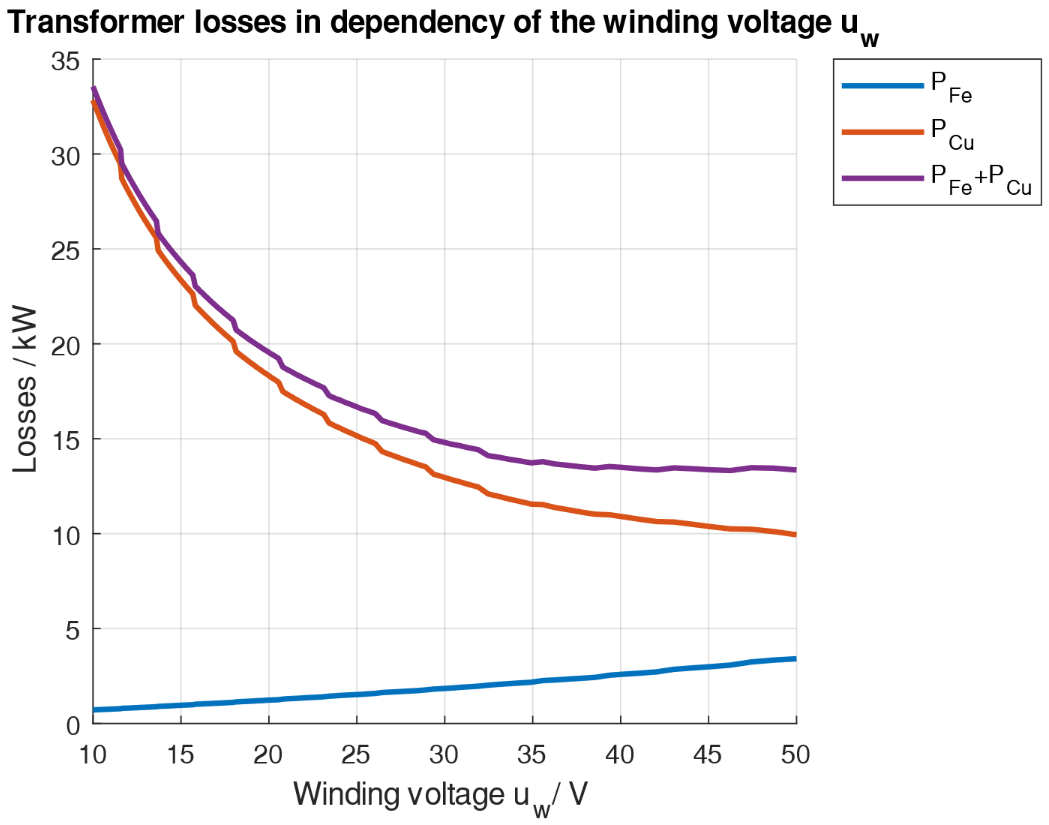
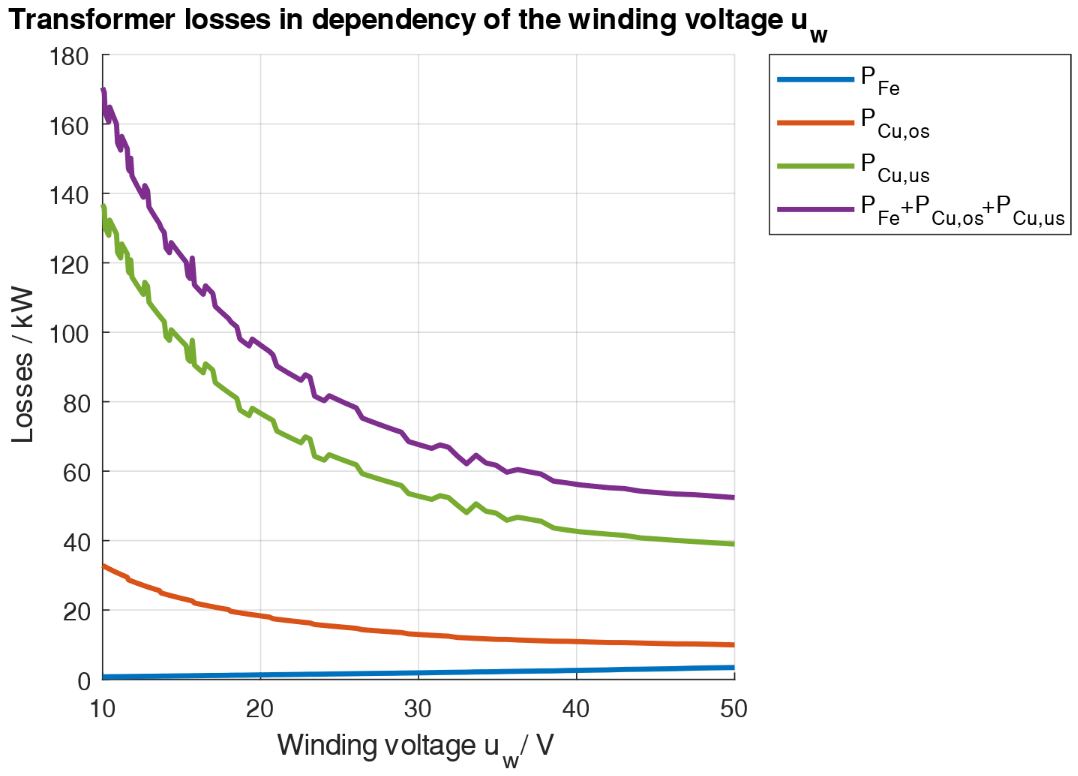
superconducting

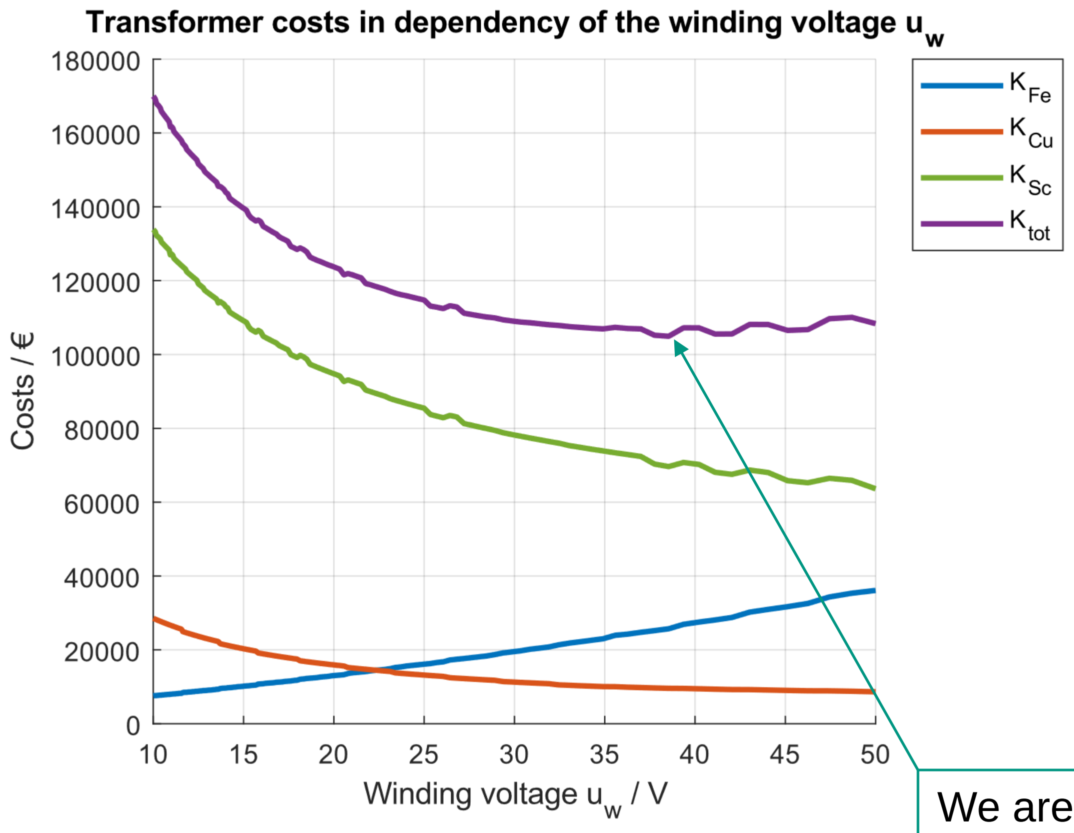


normal conducting

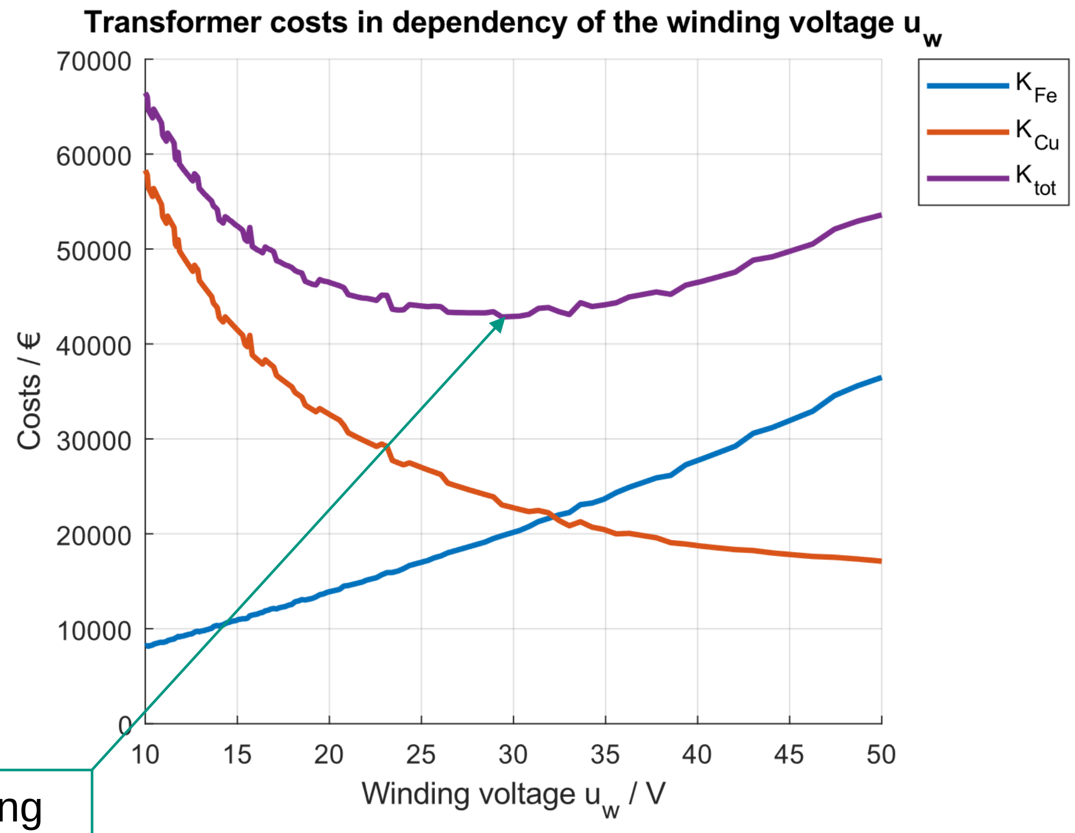


Results of the design process (4)

superconducting



normal conducting



We are looking for $u_w(K_{tot,min})$

Verification of the design process

- Checking whether all constraints are fulfilled
  - Dimensions? ✓
  - Total mass? ✓
  - Relative short-circuit voltage? ⚡
    - The relative short circuit voltage u_k of 40% can´t be reached at certain winding voltages

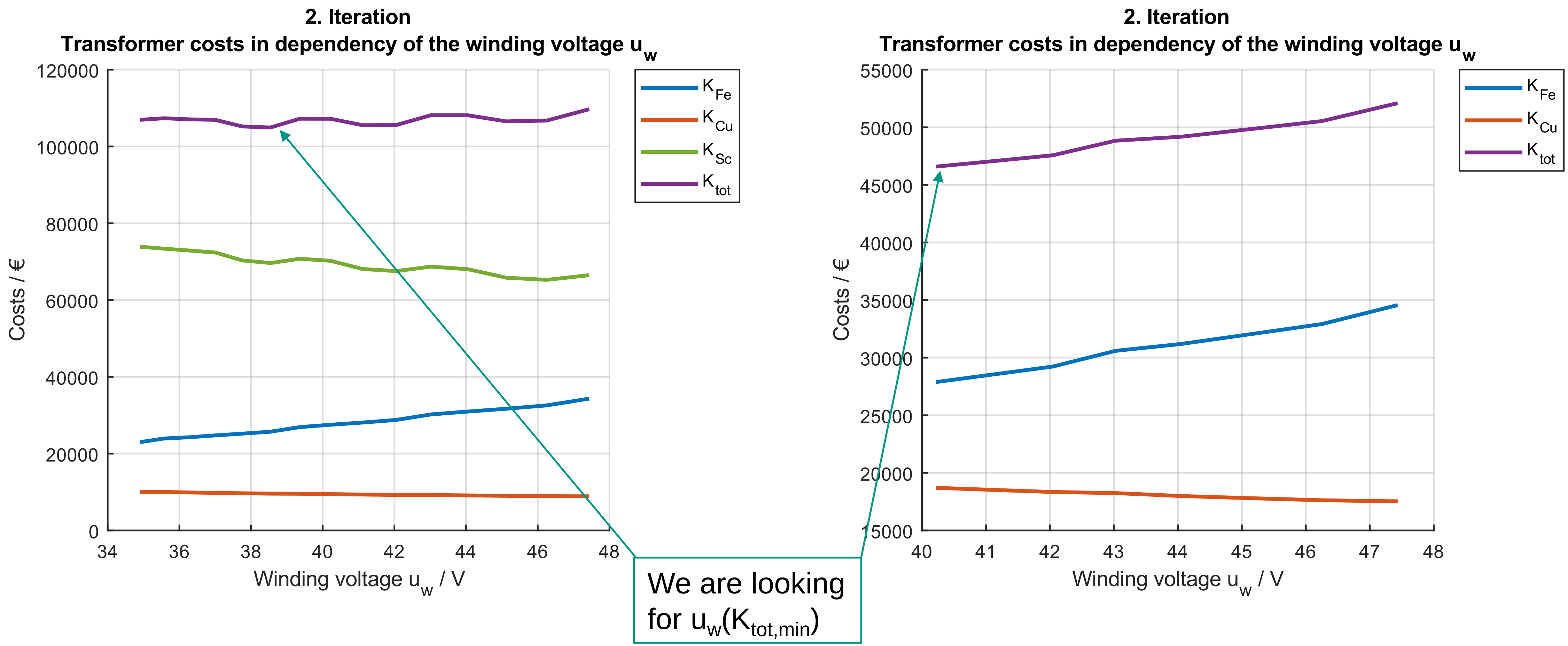
The range of 10V < u_w < 50V was empirically set at the beginning of the design process.

→ **A second iteration of the design process is needed, to eliminate winding voltages, that do not fulfill the constraints**

Results of the second iteration

superconducting

normal conducting



Comparison of Designs
superconducting vs. normal conducting

		superconducting	Normal conducting
Winding voltage	u_w	38,5417 V	41,1111V
Number of turns	w_{os}	649 (n_{layer}=13, n_h=50)	608 (n_{layer}=12, n_h=51)
	w_{us}	48 (n_{layer}=8, n_h=6)	45 (n_{layer}=8, n_h=6)
Length (max. 3400 mm)	h_y	1337 mm	1372 mm
Width (max. 2500 mm)	h_x	1159 mm	1332 mm
Height (max. 700 mm)	h_z	644 mm	666 mm
Mass (max. 6500 kg)	m	3414 kg (+X)	4406 kg
Efficiency	$\boldsymbol{\eta}$	**99,73%**	**98,89 %**
Total loss	P_V	13,390 kW	55,639 kW
Copper loss	$P_{V,Cu}$	11,024 kW	52,944 kW
Iron loss	$P_{V,Fe}$	2,366 kW	2,6954 kW
Costs of superconductors	K_{sl}	70.949 €	———
Costs of copper	K_{cu}	9.584 €	18.519 €
Costs of iron	K_{fe}	25.051 €	28.539 €
Total costs	$\boldsymbol{K_{tot}}$	**105.584 €**	**47.058 €**

Conclusion:
- Efficiency can be increased by approx. 1%
- Total mass can be reduced by approx. 1000kg
- Total costs are beeing doubled

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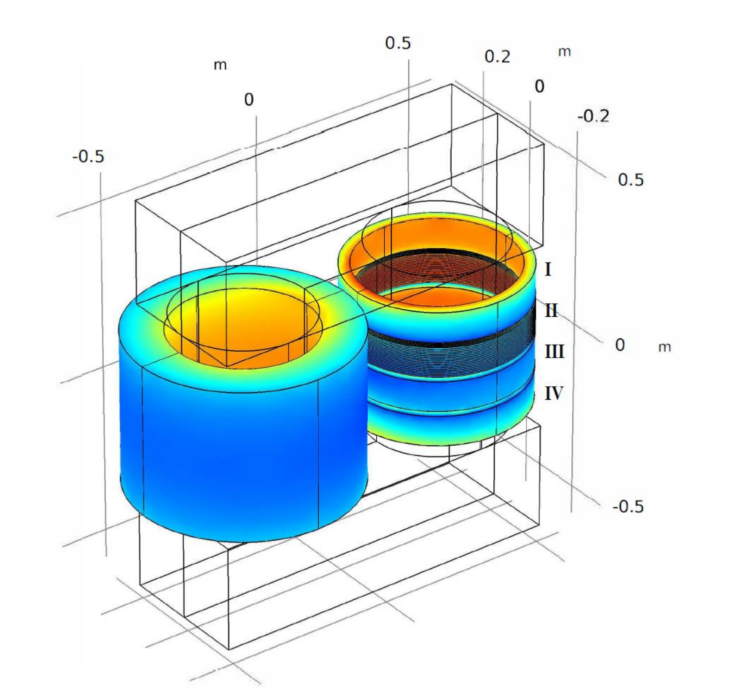
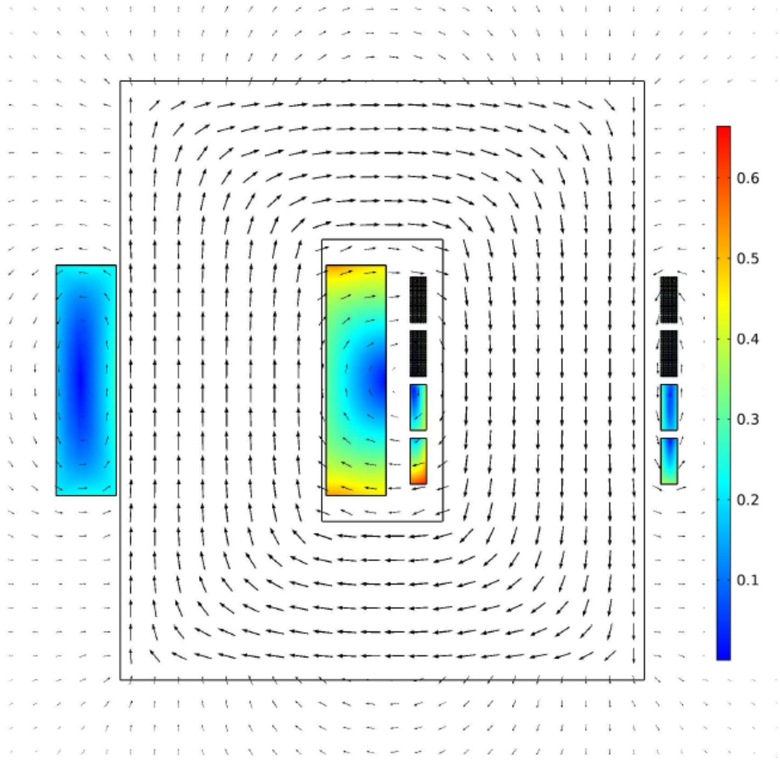
Superconducting Railway-Transformer

Additional Slides about AC-Loss

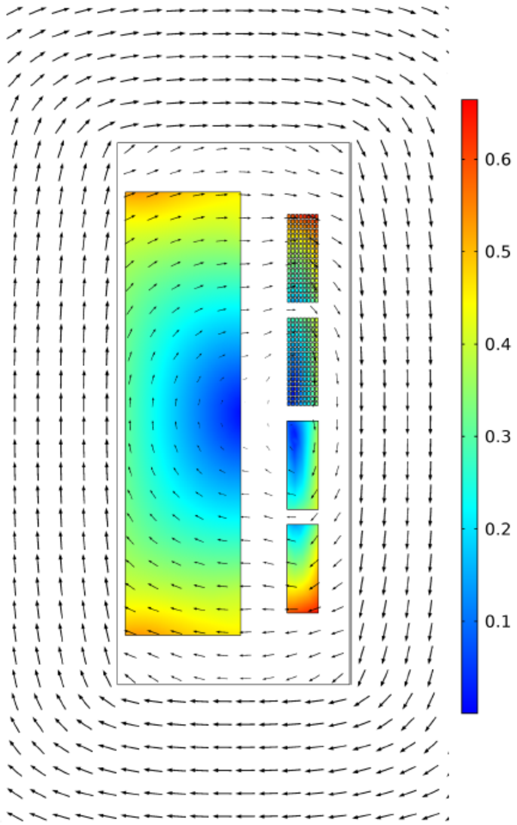
AC-Loss Calculation: Magnetic Flux Density



3D Image of the Transformer

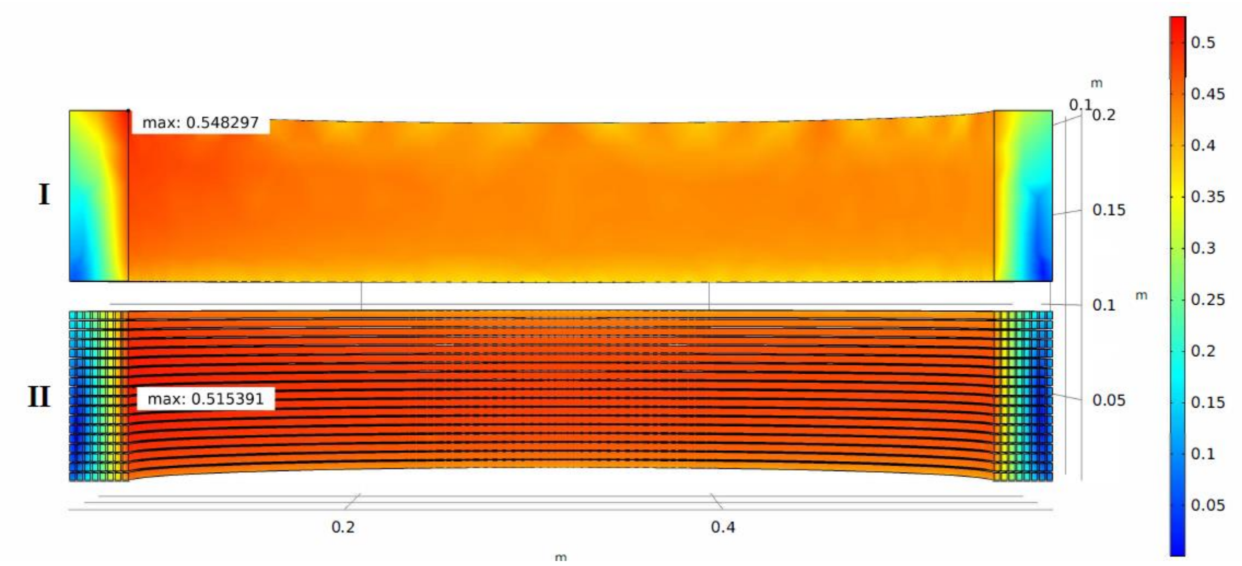


Cross-section of the Transformer

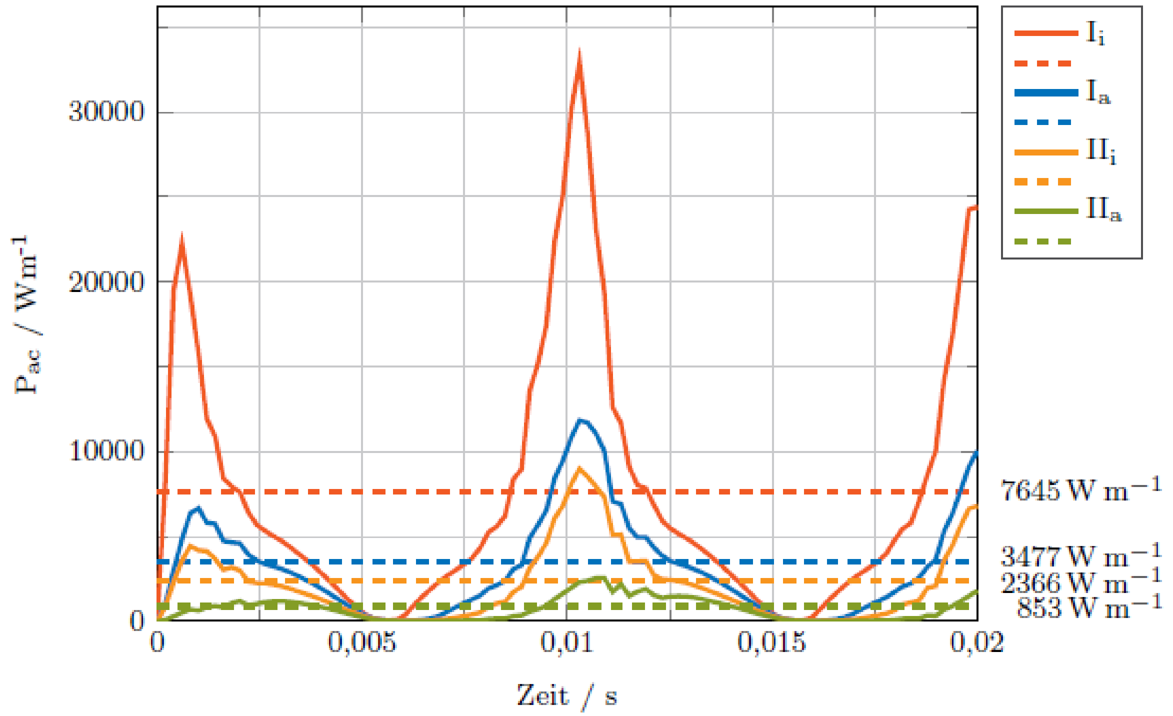


Enlarged cross-section

AC-Loss Calculation: Field Impact on different winding sections



Detailed View of the two top windings

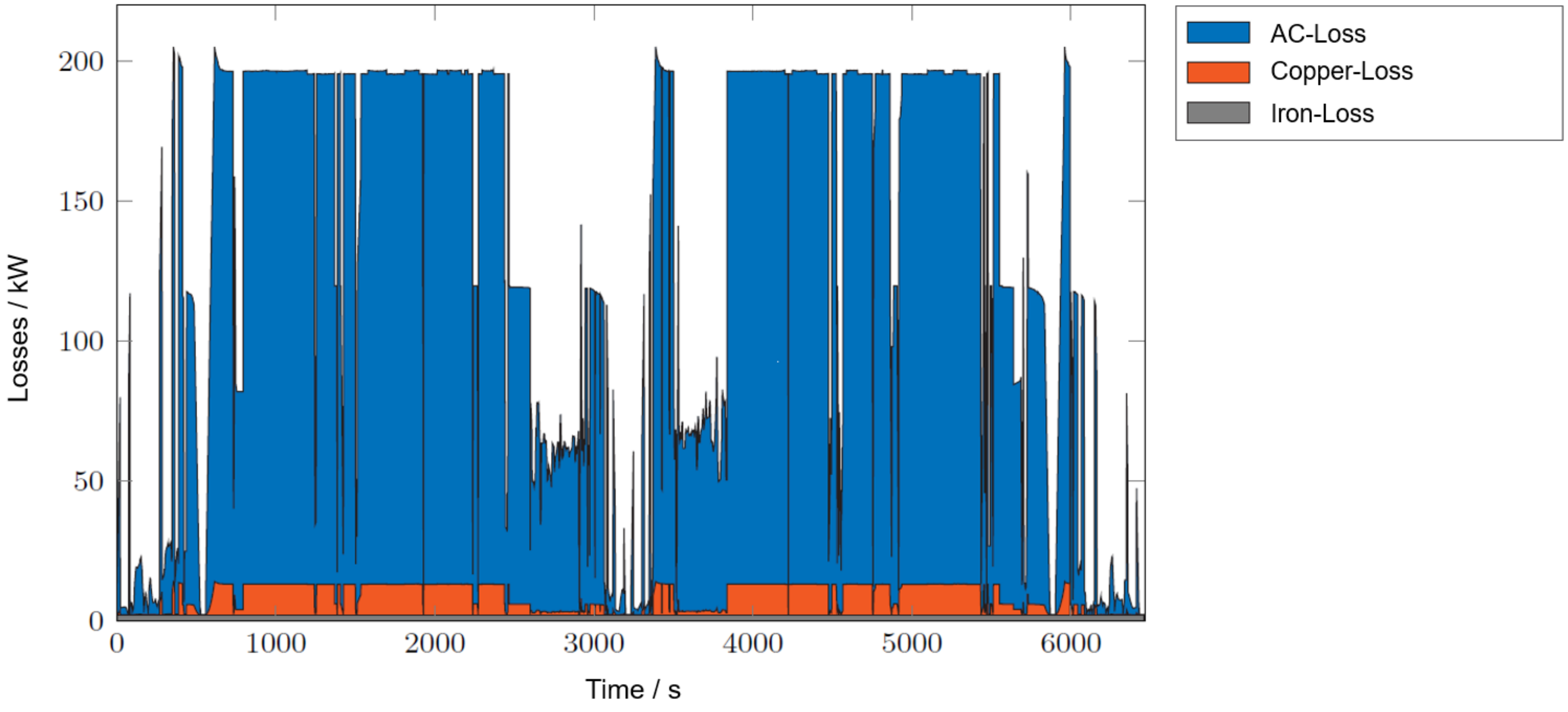


AC-Loss in different winding sections

Losses over Load

Load	**30%**	**50%**	**70%**	**100%**
Copper-Loss	0,94 kW	2,70 kW	5,53 kW	11,02 kW
Iron-Loss	2,43 kW	2,43 kW	2,43 kW	2,43 kW
AC-Loss (cold)	(5,54 kW)	9,68 kW	13,19 kW	18,39 kW
AC-Loss (warm)	55,40 kW	96,80 kW	131,90 kW	183,90 kW
Total Losses	**58,77 kW**	**101,93 kW**	**139,68 kW**	**197,35 kW**

Losses over driving profile



Losses over complete driving profile